IEEE 6th CAS Symp. on Emerging Technologies: Mobile and Wireless Comm.
Shanghai, China, May 31~June 2, 2004

# MIMO-OFDM Channel Estimation in the Presence of Carrier Frequency-Offset

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***Abstract*—A new algorithm for joint channel and carrier frequency-offset (CFO) estimation in MIMO-OFDM using a training sequence is proposed. This algorithm is developed based on a maximum likelihood (ML) criterion, and jointly estimates the CFO and all frequency-selective channel parameters in time domain. To evaluate the performance of the algorithm, the Cramer-Rao Bound (CRB) for the problem is calculated. Computer simulations show that the proposed algorithm can meet the CRB in certain Signal-Noise Ratio (SNR).**

***Keywords*—MIMO-OFDM, CFO, channel estimation, CRB, ML**

## I. INTRODUCTION

Multiple-input multiple-output (MIMO), with its ability to improve the capacity [1], is expected to play an important role in future wireless communication. However, in frequency-selective fading channel, complicated channel equalization technique is required in MIMO to combat the intersymbol interference (ISI).

In orthogonal frequency division multiplexing (OFDM), the entire channel is divided into many narrow parallel subchannel, thereby increasing the symbol duration and reducing or eliminating the ISI by multipath.

To reduce the realization complexity of MIMO, the scheme of applying OFDM technique to this system has been proposed in recent years. Recent laboratory test and field trial results show the "encouraging" performance of the MIMO-OFDM [2]. However, the inter-carrier interference (ICI) caused by carrier-frequency offset between the transmitter and receiver local oscillators severely degrades the performance of OFDM, furthermore, with the influence of channel, the theoretical benefits of MIMO-OFDM system may not be fully achieved.

The channel estimation problem for MIMO-OFDM was first studied by Li [3]. A corresponding simplified algorithm was presented in [4]. Timo et al [5] proposed channel tracking and equalization method in MIMO-OFDM stemming from Kalman filtering. In the literatures, most channel estimation methods assume perfect CFO knowledge. Mody and Stuber [6] discuss a time and frequency synchronization technique for MIMO-OFDM system. Honan and Tureli [7] also proposed a blind algorithm for CFO estimation in MIMO-OFDM system. However, the algorithms aforementioned did not consider the channel estimation in the presence of CFO. Besson and Stoica [8] address the joint CFO and channel gains estimation problem for MIMO flat fading channel using a training sequence.

This paper proposed a new joint channel and CFO estimation algorithm in MIMO-OFDM systems using a training sequence. The algorithm is developed based on a ML criterion. Its performance can meet CRB in certain SNR. Furthermore, because of time domain processing, the algorithm is also suitable to MIMO systems in frequency-selective channel. The essential difference between the proposed method and Besson's method [8] is that Besson's method is applied to flat fading channel and proposed method can deal with the frequency-selective channel.

The organization of this paper is as follows. Section II presents the system model and the proposed algorithm is developed in Section III. The CRB is given in Section IV. Simulation results and analysis are presented in Section V, and the paper is concluded in Section VI.

## II. SYSTEM MODEL

The MIMO-OFDM transmission model used in this paper is presented in Fig.1. [5]. A 2-transmit / 2receive antenna configuration is considered. The extension to any transmit/receive antenna case is a straightforward generalization.

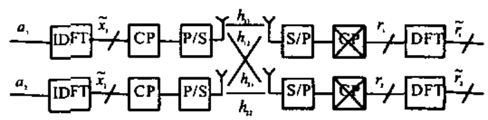



Fig1. MIMO-OFDM transmission

The $k^{th}$ modulated OFDM block of the $i^{th}$ transmit antenna is written as $\tilde{x}_i(k) = F_N a_i(k)$, where $F_N$ is $N \times N$ inverse discrete Fourier transform (IDFT) matrix, $N$ being the total number of subcarriers and $a_i(k)$ is the $N \times 1$ complex symbol vector sent from antenna $i$. We assume there is a carrier frequency offset between the transmitter and receiver, $\bar{f}$ is the carrier frequency offset normalized by sampling frequency. After the removal of the cyclic prefix (CP) of size G and directly generalization of signal model in [5], the receive signal with CFO is expressed as:

$$\begin{bmatrix} r_1(k) \\ r_2(k) \end{bmatrix} = \begin{bmatrix} C_0(\bar{f}) & 0 \\ 0 & C_0(\bar{f}) \end{bmatrix} \begin{bmatrix} \tilde{H}_{11}(k) & \tilde{H}_{21}(k) \\ \tilde{H}_{12}(k) & \tilde{H}_{22}(k) \end{bmatrix} \begin{bmatrix} \tilde{x}_1(k) \\ \tilde{x}_2(k) \end{bmatrix} \xi + \begin{bmatrix} w_1(k) \\ w_2(k) \end{bmatrix} \quad (1)$$

or, in more compact form:

$$r(k) = C(\bar{f}) \tilde{H}(k) \tilde{x}(k) \xi + w(k), \quad (2)$$

where $r_j(k)$ is the $k^{th}$ receive block of size $N \times 1$ at antenna $j$ ; $C_0(\bar{f}) = diag[1, e^{j2\pi\bar{f}}, \cdots, e^{j2\pi\bar{f}(N-1)}]$ ; $\xi = e^{j2\pi\bar{f}((k-1)N+kG)}$; $\tilde{H}_{ij}$ is the circulant matrix constructed by channel taps $\{h_{i,j,l}\}_{l=0,1,\ldots,L-1}$, with the $(r,l)$th entry given by $h_{i,j,(r-l)}$ mod $N$. The maximum channel length is assumed to be $L$, and channel impulse responses are considered to be uncorrelated with each other. $w(k)$ is assumed to be circular white Gaussian with zeros-mean and variance $\sigma^2$.

Since $\tilde{H}_{ij}$ are circulant matrice, they can be diagonalized by IDFT and DFT operation. However, as $C_0(\bar{f})$ exists, after discrete Fourier transform (DFT) of $r_j(k)$ we obtain:

$$\begin{aligned}\tilde{r}_j(k) &= F_N^H C_0(\bar{f})\tilde{H}_{1j}(k)F_N a_1(k)\xi \\ &\quad + F_N^H C_0(\bar{f})\tilde{H}_{2j}(k)F_N a_2(k)\xi + F_N^H w_j(k) \\ &= D_{1j}(k)a_1(k)\xi + D_{2j}(k)a_2(k)\xi + \tilde{n}_j(k),\end{aligned} \tag{3}$$

where, $j = 1,2$, $\tilde{n}_j = F_N^H w_j(k)$, $F_N^H$ is DFT matrix and $(\bullet)^H$ denotes the complex conjugate transpose. Obviously, $D_{ij}(k) = F_N^H C_0(\bar{f})\tilde{H}_{ij}(k)F_N$ are not diagonal matrice, and thus introduces ICI. Now $\bar{f}$ is required to cancel $C_0(\bar{f})$ term in time domain, so as to recover diagonalization of $D_{ij}(k)$. After that, if channel information is known, the equalization can be realized in lower complexity with the help of this special matrix structure, and then detect the symbol.

## III. DERIVATION OF PROPOSED ALGORITHM

From Section II, we can see, CFO and channel parameter estimation is the key for MIMO-OFDM system to achieve higher performance and lower complexity. In this section, a ML based joint CFO and channel estimation algorithm has been proposed.

We assume channel parameters and CFO are invariant in several OFDM modulation block, and using one block as a training sequence. Since estimation can be achieved in one block, we can drop $k$ in (1) firstly, and then change (1) to the following equivalent form:

$$\begin{aligned} r &= C(\bar{f})\begin{bmatrix}\tilde{X}_1 & \tilde{X}_2 & 0_{N\times L} & 0_{N\times L} \\ 0_{N\times L} & 0_{N\times L} & \tilde{X}_1 & \tilde{X}_2\end{bmatrix}\begin{bmatrix}h_{11}\\ h_{21}\\ h_{12}\\ h_{22}\end{bmatrix} + w \\ &= C(\bar{f})\tilde{X}h + w,\end{aligned} \tag{4}$$

where, $h = [h^T{}_{11} \ \ h^T{}_{21} \ \ h^T{}_{12} \ \ h^T{}_{22}]^T$ ( $(\bullet)^T$ denotes the transpose) is constructed by stacking all the channel tap $h_{ij} = [h_{i,j,0} \ \ h_{i,j,1} \ \cdots \ h_{i,j,L-1}]^T$ up to a $4L\times 1$ vector. $\tilde{X}_i$ denotes the circulant matrix with size of $N\times L$ stacked by modulate block $\tilde{x}_i$.

$$\tilde{X}_i = \begin{bmatrix}\tilde{x}_{i,1} & \tilde{x}_{i,N} & \cdots & \tilde{x}_{i,N-L+2} \\ \tilde{x}_{i,2} & \tilde{x}_{i,1} & \cdots & \tilde{x}_{i,N-L+3} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{x}_{i,N} & \tilde{x}_{i,N-1} & \cdots & \tilde{x}_{i,N-L+1}\end{bmatrix}_{N\times L}, \tag{5}$$

Since all parameters except noise are determinant, the likelihood function of receive data is given by

$$L(r) = \frac{1}{(\pi\sigma^2)^{2N}}\exp\left[-\frac{1}{\sigma^2}(r - C(\bar{f})\tilde{X}h)^H(r - C(\bar{f})\tilde{X}h)\right], \tag{6}$$

Thus, the log-likelihood function is

$$\ln(L) = const - 2N\ln(\sigma^2) - \frac{1}{\sigma^2}\| r - C(\bar{f})\tilde{X}h\|^2 , \tag{7}$$

The estimation of $\bar{f}$ and $h$, which maximizes the likihood function above, is the solution of the following joint optimization problem

$$\min_{f,h} \| r - C(\bar{f})\tilde{X}h\|^2 , \tag{8}$$

Let $\bar{f}$ be given, we can obtain

$$h = (\tilde{X}^H\tilde{X})^{-1}\tilde{X}^H C^H(\bar{f})r , \tag{9}$$

Given $I = C(\bar{f})C^H(\bar{f})$ , let $P = \tilde{X}(\tilde{X}^H\tilde{X})^{-1}\tilde{X}^H$ , substituting (9) into (8), we can obtain:

$$\hat{\bar{f}} = \min_f \| (I - P)C^H(\bar{f})r\|^2 , \tag{10}$$

Obviously, $P$ is a projection matrix, $I$ denotes identity matrix, Equ. (10) is equivalent to the following formula:

$$\hat{\bar{f}} = \max_f \| PC^H(\bar{f})r\|^2 = \max_f [r^H C(\bar{f})PC^H(\bar{f})r] , \tag{11}$$

let,

$$J(\bar{f}) = r^H C(\bar{f})PC^H(\bar{f})r , \tag{12}$$

The resulting algorithm is summarized in the following steps:

**Step 1. The Coarse Search:** In this step, we search the maximum of $J(\bar{f}_k)$ at frequency grid $\bar{f}_k$, where $\bar{f}_k = k/MT$, $k = 0,1,\ldots, M-1$, $M$ is commonly selected as $2N$ or $4N$ [9]. In practice, the grid space can be determined flexibly according to the search range.

**Step 2. The Fine Search:** With the value $\bar{f}_k$ from step 1 as the initial value, some optimization method such as Newton method can be used to obtain the accurate estimation of $\hat{\bar{f}}$.

**Step 3. Channel Estimation:** Substituting $\hat{\bar{f}}$ into (9), the channel parameters can be obtained.

Computer simulations show that the convergent point can be met using Newton method after 2 or 3 iterations.

It's indicated that the estimated value of CFO is independent of the estimated channels, however, the estimated accuracy of $\bar{f}$ influences the final estimated accuracy of channel $h$.

## IV. THE CRB

The deterministic CRB is derived here to judge the performance of the proposed algorithm. We now construct the Fisher Information Matrix (FIM) [11] by calculating the derivative of (7) with respect to $\eta = [\tilde{f} \quad Re(\boldsymbol{h})^T \quad Im(\boldsymbol{h})^T]$, the expression for FIM is shown below

$$FIM = \frac{2}{\sigma^2}\begin{bmatrix} \mathrm{Re}(\tilde{X}^H \tilde{X}) & -\mathrm{Im}(\tilde{X}^H \tilde{X}) & -2\pi\,\mathrm{Im}(\tilde{X}^H B\tilde{X}h) \\ \mathrm{Im}(\tilde{X}^H \tilde{X}) & \mathrm{Re}(\tilde{X}^H \tilde{X}) & 2\pi\,\mathrm{Re}(\tilde{X}^H B\tilde{X}h) \\ 2\pi\,\mathrm{Im}(h^H \tilde{X}^H B\tilde{X}) & 2\pi\,\mathrm{Re}(h^H \tilde{X}^H B\tilde{X}) & 4\pi^2\,\mathrm{Re}(h^H \tilde{X}^H B^2 \tilde{X}h) \end{bmatrix} \quad (13)$$

For limited space, the derivation of FIM is omitted here.

From (13), FIM is related to the training sequence and the exact value of channel parameters. Substituting corresponding value into (13), we can numerically compute the variance of individual parameter estimate by inverting the FIM $CRB(\eta) = diag\{FIM^{-1}\}$.

Since there are too many independent channel parameters, for example, in 2-transmitter/2-receiver system, the length of channel is $L$, after the combination of real part and imagine part, there are still $4L$ CRB values, the performance evaluation of estimated channels are complicated. It is noted that all of the channel parameters are independent, so we can evaluate the performance of estimation by the average of all channel variance, which is used in the simulation of the next section.

## V. PERFORMANCE EVALUATION THROUGH SIMULATION

In this section, some simulations are conducted to evaluate the performance of the algorithm. Two MIMO-OFDM modulation blocks are used as training sequence at 2 transmit separately, and QPSK symbol modulation is employed. The additive channel noise is white Gaussian with zeros-mean. The normalized CFO is selected as 0.48. Channel parameters corresponding to different transmit or receive antennas is independent identically distributed (i.i.d.), and delay-power-spectrum function is exponential, where, $L = 5$. For each simulation, 500 Monte Carlo trials are run. The estimation performance is evaluated by Mean-Squared Error (MSE). Corresponding to CRB in Section IV, the MSE of channel is defined as

$$MSE_{ch} = \frac{1}{4L}\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{l=1}^{L} E\{|h_{i,j,l} - \hat{h}_{i,j,l}|^2\} \quad (14)$$

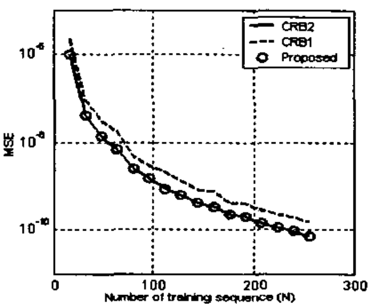
The MSE of CFO is defined as

$$MSE_{\tilde{f}} = E\{|\tilde{f} - \hat{\tilde{f}}|^2\} \quad (15)$$

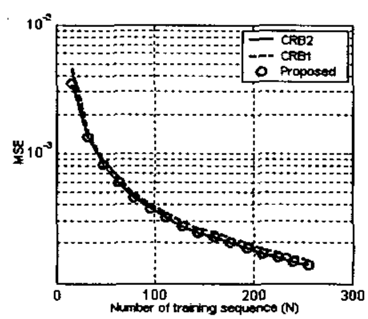
### A. Simulation 1: The influence of training number

The SNR is fixed at 15dB. Fig.2 shows the estimation performance of channel, and frequency with different numbers of training sequence respectively. Where, CRB1 is CRB of joint estimation using 2-transmit/1-receive (2-T/1-R) antennas. CRB2 is the CRB of 2-T/2-R antennas. From Fig.2, the following facts can be observed.

- The proposed algorithm is very close to CRB, even for short data samples.
- With more training sequence, the estimated accuracy is improved, but it is not improved linearly with number of training sequence N.
- The estimation performances of both channel and CFO with 2-T/2-R antennas are better than that with 2-T/1-R. In system model shown in Fig.1, every channel of receive antenna is independent, and no new information can be provided to channel estimation when receive antenna increases. So the improvement of channel estimation is due to the improvement of frequency estimation.
- In fact, CRB is related to training sequence, so the CRB plotted in simulation is the CRB corresponding to certain training sequence. Optimum CRB can be achieved by selecting optimum training sequence.




(a) CFO estimation performance(SNR=15dB)

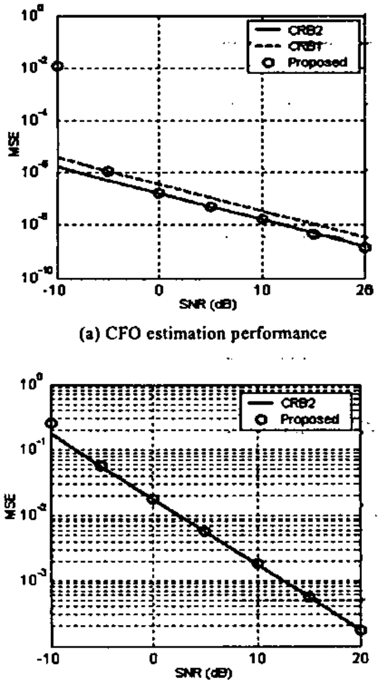



(b) Channel estimation performance(SNR=15dB)

Fig.2. The influence of number of training sequence N.

### B. Simulation 2: The influence of SNR

The influence of SNR is studied for $N = 64$. It can be seen from fig.3 that the CFO estimation MSE of the proposed algorithm is apart from the CRB when SNR is below -5dB, and the MSE almost coincides with the CRB when CRB is above -5dB, which is caused by the threshold effects [9]. Obviously, the threshold of this simulation is SNR=-5~0dB. By the influence of threshold effect of the CFO estimation, the performance of channels estimation is degrade in corresponding SNR. Furthermore, the performance of proposed algorithm is improved linearly with SNR, this point can be confirmed by the expression of FIM matrix in (13).




(b) Channel estimation performance

Fig. 3. The influence of SNR ( $N = 64$ )

## VI. CONCLUSION

This paper proposed a new joint channel and CFO estimation algorithm using a training sequence in MIMO-OFDM system, and its CRB is also derived. The algorithm is developed based on ML criterion, and can estimate CFO and all frequency-selective channel parameters. Finally, the computer simulation results show that the performance of estimation is close to the corresponding CRB at certain SNR.

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