Fast Maneuvering Target Tracking Algorithm with Multiple Passive Sensors

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Abstract: For maneuvering target tracking with multiple passive sensors in clutter environment, a novel tracking algorithm based on maximum entropy fuzzy clustering is proposed. Firstly, the interacting multiple models (IMM) approach is used to solve the maneuver problem of the target, and the false alarms generated by clutter are accommodated through maximum entropy fuzzy probabilistic data association filter (MEF-PDAF). Secondly, in order to avoid the unobservability problem of passive target tracking, a nonlinear measurement model of multiple passive sensors is formulated. Moreover, in order to reduce the computational load, the characteristic of the discrimination factor is analyzed, and the influence of the clutter density on it is also considered, which enables the algorithm eliminate those invalidate measurements. Finally, the simulation results show that the proposed algorithm has the advantages over the conventional IMM-PDAF algorithm in terms of simplicity and efficiency.

Key words: maneuvering target tracking, interacting multiple models (IMM), MEF-PDAF

1. Introduction

The key to successful maneuvering target tracking in clutter environment lies in the effective extraction of useful information about the target’s state from observations and deal with the problem arises from the uncertainty in the maneuvering command driving the target and the uncertain origin of the measurements. The underlying statistical model for our problem is a particular case of dump Markov Linear Systems (JMLS) [8, 9], which is a linear system whose parameters evolve with time according to a finite state Markov chain. JMLS are widely used in target tracking. It is well known that the exact computation of the conditional mean filtered or
smoothed state estimates involves a prohibitive computational cost exponential in the number of observations [10].

Several algorithms have been presented in the literature to address this problem, including the Interacting Multiple Model Probabilistic Data Association Filter (IMM-PDAF) algorithm, the Probabilistic Multi-Hypothesis Tracking (PMHT) algorithm, batch Expectation Maximization (EM), Markov chain Monte Carlo (MCMC) methods and the particle filtering [5]. A promising approach is the interacting multiple models (IMM) algorithm originally by Blom [6], which is based on a hybrid system description of the maneuver scenarios and the occurrence of target maneuvers is explicitly included in the kinematics equations through regime jumps. In the presence of clutter, the integration of the IMM and PDAF [7] is an efficient solution to the uncertainty of measurements origin [1]. In the tracking benchmark problem [8] designed to compare the performance of different algorithms for tracking highly maneuvering targets in the presence of electronic countermeasures, the PDA-based estimator, in conjunction with the interacting multiple model (IMM) estimator, yielded one of the best solutions. Its performance was comparable to that of the MHT algorithm [9].

Passive tracking of moving targets using only bearing or line of sight (LOS) angle measurements is a known nonlinear estimation problem. Since the problem is nonlinear, the usual approach for recursive estimation is to employ an extended Kalman filter (EKF). Because the LOS is an incomplete position observation it cannot be converted into Cartesian coordinates to allow for linear filtering as in [10]. In recursive bearings-only tracking the use of the Cartesian coordinate EKF' has been shown achieving erratic estimation results and unstable behavior [1], even without the detrimental effects caused by the presence of false detections or clutter. Furthermore, the azimuth and elevation measurements of a passive sensor do not allow an instantaneous range determination. In order to solve this problem, there have usually two solutions. First, the passive sensor platform is allowed to move freely, it is possible to recover range observability by selecting an appropriate path for the platform. In some applications the sensor platforms have very slow mobility compared with the target dynamics and this solution is not feasible. A solution is then to use several passive sensors and fuse their information in some way to estimate the range.

Dufour and Michel [12] proposed an extension version of IMM-PDAF to track a 3D
maneuvering target with two passive sensors in clutter environment. The contribution of their work is twofold. Firstly a novel application of the IMM algorithm is studied where passive-only sensors are fused for tracking a target maneuvering in three dimensions. Secondly, several accurate models of target motion are proposed to improve the performance. The main shortcoming of their method is the heavy computational load, particularly in heavy clutter environment, thus being difficult for practical application. Recently, we propose a target tracking method based on maximum entropy fuzzy clustering, which has the same performance as that of PDAF while with lower computational load [13]. Inspired from literatures [12,13], a novel algorithm based on Maximum entropy fuzzy clustering is proposed for real time maneuvering target tracking in this paper.

The remainder of the paper is organized as follows. The problem formulation is presented in Section 2. The maximum entropy clustering to single target tracking is described and a novel maneuvering target tracking is introduced in Section 3. Simulation results of the performance comparison of the existing algorithms are presented in Section 4. Finally, some conclusions are provided in Section 5.

2. PROBLEM FORMULATION

2.1 System setup

Given a system

\[ x_k = F(c_k)x_{k-1} + v_k \quad (1) \]
\[ y_k = h(x_k) + w_k \quad (2) \]

where \( x_k \in \mathbb{R}^n \) is the dynamical state of the system in model \( c_k \), \( c_k \in \{1,2,\ldots,M\} \) is the model state of the system (e.g., \( c_k = 1 \) when the target moves at constant speed and \( c_k = 2 \) when the target accelerates). The process noise vector \( v_k \) and the measurement noise vector \( w_k \) are independent, zero mean noise with known covariance \( Q(k) \) and \( R(k) \) respectively.

The mode transition of the system is modeled by a Markov chain with

\[ \pi_{ij} = P(c_{k+1} = j | c_k = i) \quad \forall i, j \in M \quad (3) \]

or, in vector form

\[ p_{k+1} = \Pi_k p_k \]
2.2 Target Motion Models

Suppose that the state vector of the target is \((x(k), y(k), z(k), \dot{x}(k), \dot{y}(k), \dot{z}(k))\). In this paper, the interacting multiple model approach is used to solve the maneuver problem of the target. Three target motion models are used:

**Model 1:** Constant Velocity Motion

The state transition matrix and the process noise covariance matrix are defined by

\[
A = \begin{bmatrix}
1 & 0 & 0 & T & 0 & 0 \\
0 & 1 & 0 & 0 & T & 0 \\
0 & 0 & 1 & 0 & 0 & T \\
0 & 0 & 0 & 1 & 0 & T \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[
Q = \begin{bmatrix}
\frac{1}{4}T^4 & 0 & 0 & \frac{1}{2}T^2 & 0 & 0 \\
0 & \frac{1}{4}T^4 & 0 & 0 & \frac{1}{2}T^2 & 0 \\
0 & 0 & \frac{1}{4}T^4 & 0 & 0 & \frac{1}{2}T^2 \\
\frac{1}{4}T^4 & 0 & 0 & T^2 & 0 & 0 \\
0 & \frac{1}{4}T^4 & 0 & 0 & T^2 & 0 \\
0 & 0 & \frac{1}{4}T^4 & 0 & 0 & T^2 \\
\end{bmatrix}
\]

**Model 2:** Constant Turn Motion

The state transition matrix is

\[
A = \begin{bmatrix}
1 & 0 & 0 & \frac{\sin(w)}{w} & \frac{\cos(w) - 1}{w} & 0 \\
0 & 1 & 0 & \frac{1 - \cos(w)}{w} & \frac{\sin(w)}{w} & 0 \\
0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & \cos(w) & -\sin(w) & 0 \\
0 & 0 & 0 & \sin(w) & \cos(w) & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

where \(w\) is a constant angular rate. The process noise covariance matrix is the same as Model 1.

**Model 3:** For \(w < 0\) describes a clockwise turn, and Model 3 is its natural counterpart for a counterclockwise turn \(w > 0\).

3. PROPOSED MANEUVERING TARGET TRACKING METHOD

3.1 Maximum Entropy Fuzzy Clustering

To be specific, suppose that an N-point data set \(\{x_i, i = 1, 2, \ldots, N\}\) is related to one of the clusters \(\{c_j, j = 1, 2, \ldots, c\}\). The clustering process can be formulated as an optimization problem
and the corresponding cost function to be minimized is defined as

\[ E = \sum_{i=1}^{N} \sum_{j=1}^{c} u_{ij} \cdot d(x_i, c_j) \]  

(7)

where \( d(x_i, c_j) \) is the squared Euclidean distance between the given data point \( x_j \) and the cluster center \( c_i \). The probabilistic constraint is that the summation of all \( u_{ij} (j=1,2,\cdots,c) \) must be equal to one, i.e.

\[ \sum_{j=1}^{c} u_{ij} = 1 \quad \forall \quad u_{ij} \in [0,1] \]  

(8)

According to the information theory, the maximum entropy principle (MEP) is the most unbiased prescription to choose the values of membership \( u_{ij} \), for which the Shannon entropy, i.e., the expression

\[ H = H(u_{ij}) = -\sum_{i=1}^{N} \sum_{j=1}^{c} u_{ij} \ln u_{ij} \]  

(9)

is maximized under the constraints Eq.(7) and Eq.(8). Using the Lagrange multiplier method, the objective function can be defined as

\[ J(U,C) = -\sum_{i=1}^{N} \sum_{j=1}^{c} u_{ij} \ln u_{ij} - \sum_{i=1}^{N} \sum_{j=1}^{c} \alpha_i u_{ij} \cdot d(x_i, c_j) + \sum_{i=1}^{N} \sum_{j=1}^{c} \lambda_j (\sum_{j=1}^{c} u_{ij} - 1) \]  

(10)

By maximizing Eq (9), the membership function of data \( x_i \) belonging to cluster center \( c_j \) is derived as

\[ u_{ij} = \frac{e^{-\alpha_i d(x_i, c_j)}}{\sum_{k=1}^{c} e^{-\alpha_i d(x_i, c_k)}} \]  

(11)

where \( \alpha_i \) and \( \lambda_j \) are the Lagrange multipliers. By varying \( \alpha_i \), it adjusts the value of membership of data point \( x_i \) with its nearest cluster center \( c_j \) as compared to that of other cluster centers. Thus, we call \( \alpha_i \) as “discriminating factor”. How to choose a proper value of \( \alpha_i \) has been discussed [14, 15].

3.2 Maximum Entropy Fuzzy Clustering for Target Tracking

From Eq. (8), it shows that the summation of the values of them is equal to one. Since the membership value is also nonnegative, the membership degree of each data point to cluster centers can be regarded as the probability that the data point is clustered by cluster centers [14]. This makes it \( (u_{ij}) \) a latent candidate and an alternative to \( \beta_j \) in PDAF. Suppose there is a data set
with \( m \) measurements \((x_1, x_2, \cdots, x_m)\), in an n-dimensional space, and a given target \( T \). In order to apply maximum entropy fuzzy clustering to target tracking, the expression of Eq.(8) can be modified as

\[
\sum_{j=1}^{m} u_j = 1 \quad \forall u_j \in [0, 1]
\]  

(12)

where \( m \) denotes the number of validate measurements, \( u_j \) can be viewed as the probability that the measurement \( z_j \) belongs to the cluster center \( c \). The corresponding cost function is defined as

\[
E = \sum_{j=1}^{m} u_j \cdot d(x_j, c)
\]  

(13)

According to the maximum entropy fuzzy clustering principle described above, the corresponding main objective function \( J(U, C) \) to be maximized by the Lagrange method is defined as

\[
J(U, C) = -\sum_{j=1}^{m} u_j \ln u_j - \alpha \sum_{j=1}^{m} u_j \cdot d(x_j, c) + \lambda (\sum_{j=1}^{m} u_j - 1)
\]  

(14)

where \( \alpha \) and \( \lambda \) are the Lagrange multipliers, \( d(z_j, c) \) may be the squared Euclidean distance (or the Mahalanobis distance) between the measurement \( z_j \) and the cluster center \( c \). The final probabilities are

\[
u_j = \frac{e^{-d(x_j, c)}}{\sum_{j=1}^{m} e^{-d(x_1, c)}} \quad j = 1, 2, \cdots m
\]  

(15)

The probabilities given by Eq. (15) are the least biased judgment possible for \( u_j, i = 1, 2, \cdots m \). This claim is justified in [14]. It is noted that the membership degree \( u_j \) is very approximate to the association probability \( \beta_j \) in PDAF [7]. The only difference between them is the exponential terms. So if a proper value of \( \alpha \) is selected, the membership degree can completely replace the association probability with the same or better performance. The association probability can be defined as

\[
\beta_j = u_j
\]  

(16)

Furthermore, if there is no measurement within the validation gate of target \( T \), the update state estimation will be simply the previous one, i.e.

\[
\hat{x}(k | k) = \hat{x}(k | k - 1)
\]  

(17)
Another important question we have not answered yet is how to determine the cluster center $c$. In order to avoid being trapped into the local maximum, a good initial guess is highly preferable. For this purpose, a good candidate for the center $c$ is provided by the predicted measurement, that is,

$$C = H(k)F(k) \hat{x}(k-1|k-1)$$  \hspace{1cm} (18)

3.3 IMM-MEFPDAF

According to the descriptions above, the MEFPDAF developed in this paper consists in considering the PDAF algorithm where the association probabilities are substituted by membership grades provided by maximum entropy fuzzy clustering. In this section, the IMM-MEFPDAF is proposed that the interacting multiple models and the maximum entropy fuzzy probabilistic data association are integrated and the basic structure of the algorithm is the same as the IMM-PDAF. A detail derivation of the algorithm structure can be founded in [6] while the new algorithm is not described here.

**Step 1: Interaction**

Mixing probabilities

$$\mu_{jk}(k-1|k-1) = \frac{1}{c_j} p_{ji} \mu_j(k-1)$$  \hspace{1cm} (19)

Normalizing factors

$$c_j = \sum_{j=1}^{M} p_{ji} \mu_j(k-1)$$  \hspace{1cm} (20)

Mixing initial state estimated in mode $j$

$$\hat{X}^{0i}(k-1|k-1) = \sum_{j=1}^{M} X_j(k-1|k-1) \mu_{jk}(k-1|k-1) \quad i = 1,2,\ldots,M$$  \hspace{1cm} (21)

The corresponding state covariance is

$$P^{0i}(k-1|k-1) = \sum_{j=1}^{M} \mu_{jk}(k-1|k-1) \left[ P^j(k-1|k-1) + \left[ X^j(k-1|k-1) - \hat{X}^{0i}(k-1|k-1) \right] \right]$$  \hspace{1cm} (22)

**Step 2: Prediction**

With $\hat{X}^{i}(k-1|k-1)$ and its covariance $P^i(k-1|k-1)$, one computes the predicted state $\hat{X}^i(k|k-1)$ and its covariance $P^i(k|k-1)$, and the predicted measurement $\hat{z}^i(k)$ and the corresponding covariance $S^i(k)$.

$$\hat{X}^i(k|k-1) = F(k-1)\hat{X}^{0i}(k-1|k-1)$$  \hspace{1cm} (23)

$$P^i(k|k-1) = F(k-1)P^{0i}(k-1|k-1)F(k-1)^T + Q^i(k-1)$$  \hspace{1cm} (24)
where $F(k−1), Q^j(k−1)$ and $R^j(k)$ are from (1) ~ (2). In a passive sensor system, the measurement equation is nonlinear, so the nonlinearity $y_k = h(x_k) + w_k$ must be linearized. $H_j(k)$ is the Jacobian matrix of $h(x_k)$.

$$H_j(k) = \frac{\partial h(x_k)}{\partial x_k}$$

**Step 3: Measurement Validation**

The measurement $z'_j(k)$ is validated if and only if

$$\left( z'_j(k) - \bar{z}'(k) \right) S(k) \left( z'_j(k) - \bar{z}'(k) \right) < g$$

where $S(k)$ denotes the largest among the model conditional covariances. $g$ (g-sigma gate) can be set by $P_g$ is the probability to validate a target measurement within the gate.

**Step 4: Model Probabilities Update**

The model probabilities are updated by

$$\mu_j = \frac{1}{c} \Lambda_j(k) \sum_{i \in M} p_j \mu_i(k-1) = \frac{1}{c} \Lambda_j(k) \pi_j$$

where $\pi_j$ is the expression from Eq.(20) and $c$ denotes the normalization constant. $\Lambda_j(k)$ is the likelihood function that is the joint probability density function of the innovations, written as [1]:

$$\Lambda_j(k) = p \left( Z(k) \mid M_j, Z(k-1) \right)$$

$$= p \left( v'_1(k), \ldots, v'_m(k) \mid m(k), Z(k-1) \right)$$

$$= \left( \sum_{i=1}^{m(k)} e_i(k) \right) P_d P_g m(k)^{-1} V_k^{-m(k)+1}$$

where $P_d$ is the probability to detect the target, $m(k)$ is the number of the measurements at time $k$. $V_k$ is the volume of the validation region at time $k$ for model $j$.

**Step 5: Filtering and Fusion**

With the maximum entropy fuzzy probabilistic data association filter, the update of model $i$ is computed as follows:
\[
\hat{X}^i(k | k) = \hat{X}^i(k | k - 1) + \sum_{j=1}^{K} \beta_j^i(z_j^i(k) - \hat{z}^i(k))
\]
(31)

\[
P^i(k | k) = P^i(k | k - 1) - W^i(k | k)S^i(k)W^i(k | k)
\]
(32)

where \(\beta_j^i\) denotes the association probability can be computed from (15)-(16).

Finally, the combination of the model-conditioned estimates is done.

\[
\hat{X}(k | k) = \sum_{i=1}^{M} \mu_i \hat{X}^i(k | k)
\]
(33)

\[
P(k | k) = \sum_{i=1}^{M} \mu_i \left[ P^i(k | k) + \left[ \hat{X}^i(k | k) - \hat{X}(k | k) \right] \left[ \hat{X}^i(k | k) - \hat{X}(k | k) \right] \right]
\]
(34)

### 3.4 Discrimination Factor \(\alpha_{(opt)}\)

A key issue remains unanswered is how to choose an optimal value for \(\alpha\). The optimal value \(\alpha\), denoted as \(\alpha_{(opt)}\), is derived subject to the minimization of the expected distortion given in Equation (7). Suppose that the number of validation measurements \(m_k\), is given, further research reveals that the cost function \(E\) is always monotonically and exponentially decreasing with the increment of the value of \(\alpha\) for any given data sample set \(\{x_i\}\) [14,15]. Due to the monotony, the optimal membership value \(u_i (i = 1, 2, \cdots, m)\) is reached only when \(\alpha \rightarrow \infty\). But in fact, the cost function \(E\) is also decreases exponentially with the increase in \(\alpha\), so when the discriminating factor \(\alpha\) reaches some value, the vale of \(E\) will reach a saturation state, and further increase in \(\alpha\) will only change \(E\) very slightly. At this time, it is thought that \(\alpha\) has reached the optimal value.

A detailed derivation of discrimination factor \(\alpha_{(opt)}\) can be founded in [15] while it is just briefly described here.

\[
\alpha_{(opt)} = -\frac{1}{d_{(\text{opt})}} \ln(e)
\]
(35)

In practice, the choice of an appropriate value for \(\alpha\) is depended on specific applications. To a specified problem, it is interesting to note that the larger the value of \(\alpha\) is the larger the probability of the measurements closed to the cluster center. It is notice that smaller the distance larger the corresponding probability, and greater the number of returns relating to the target higher the probability of target loss. So it is natural to require that when the density of clutter is very slow, \(\alpha\) should be large, vice versa. That is, the value of \(\alpha\) is inversely proportional to the density of
clutter $\lambda$, thus we define

$$\alpha = \frac{\eta}{\lambda \cdot d_{\min}}$$  \hspace{1cm} (36)$$

where $\eta$ is a positive constant, $\eta \in [0,1]$.

On the other hand, to a known value $\alpha$, the probability $u_i$ approximate exponentially decreases with the increment of the value of $d$. This property is important for the proposed algorithms to reduce the computational cost. When the distance between the measurement and the cluster center is sufficiently large, the measurement can be viewed as invalidated even though it may be fall in the target validation region.

Substituting $\alpha$ in Eq (15) with Eq (36), it follows that

$$u_i = \frac{e^{-\eta \cdot \lambda \cdot d_{\min} (x_i, C)}}{\sum_{j=1}^{m} e^{-\eta \cdot \lambda \cdot d_{\min} (x_j, C)}}$$  \hspace{1cm} (37)$$

Thus,

$$u_{i(\text{max})} = \frac{e^{-\eta \cdot \lambda \cdot d_{\min} (x_i, C)}}{\sum_{j=1}^{m} e^{-\eta \cdot \lambda \cdot d_{\min} (x_j, C)}}$$  \hspace{1cm} (38)$$

From Eq (38), it shows that the numerator is a constant and independent on $d(x_i, C)$, while the dominator can be approximately considered to be normalization constant. So the maximum association probability value $u_{i(\text{max})}$ can also be approximately considered to a constant. Therefore, when $u_{i(j,\text{max})}/u_{i(\text{max})} \leq \xi$ ($\xi$ is a sufficiently small positive constant, such as $\xi = 10^{-3}$ or less), the $j$ th measurement has only a slight effect on the update of state estimation of the target. In order to reduce the computational load of the algorithm, the $j$ th measurement can be regarded as a invalidate measurement. Thus, another threshold that is determined to select the set of measurements associated with the target was given as follows
In Eq (39), it shows that only the distance between the \( j \) th measurement and the cluster center \( c \) is less than \( D_{\text{max}} \), the \( j \)th measurement can be considered being a validate measurement. Thus, we call \( D_{\text{max}} \) as “maximum validate distance”. Furthermore, because the minimum distance \( d_{\min} \) is different at each sampling time, the maximum validate distance \( D_{\text{max}} \) can be adjusted automatically according to the distribution of the measurements. But the reduction of computational cost is at the cost of the slight degradation of the performance of the target estimated. Therefore, the size of threshold can be adjusted according to the different application cases.

4. SIMULATION RESULTS

In this section, the performance of the proposed algorithm is evaluated and compared with the conventional IMMPDAF algorithm [12].

4.1 Observation Model

In a single passive sensor, when the sensor is allowed to move freely, it is possible to recover range observability by selecting an appropriate path. For a stationed multiple passive sensor system, in order to avoid the observability problem of measurements, the range must be acquired through multiple passive sensors. In this paper, multiple passive sensors are considered, and the observation equation is defined as

\[
Z(k) = H(X(k)) + W(k)
\]

where

\[
Z(k) = \left[ z_1(k)^T \ z_2(k)^T \ldots \ z_n(k)^T \right]^T = \left[ (\alpha_1, \beta_1)^T \ (\alpha_2, \beta_2)^T \ldots (\alpha_n, \beta_n)^T \right]^T
\]
\[ H(X(k)) = [H_1(X(k))^T \ H_2(X(k))^T \cdots \ H_n(X(k))^T]^T \] (42)

\[ W(k) = [w_1(k)^T \ w_2(k)^T \cdots \ w_n(k)^T]^T \] (43)

where \( H_i(X(k)) \) is the nonlinear equation, \( W(k) \) denotes the measurement noise vector. We assume that the angles errors \( w_i(k) \) \((i = 1, 2, \cdots, n)\) are mutual independent and the measurement noise \( W(k) \) has a diagonal covariance \( R(k) \)

\[ R(k) = E \left[ W(k)W(k)^T \right] = \text{diag}[\sigma^2_{a_1}, \sigma^2_{\beta_1}, \ldots, \sigma^2_{a_n}, \sigma^2_{\beta_n}] \] (44)

where \( \sigma^2_{a_i} \) \((i = 1,2,\ldots,n)\) is the azimuth error of the sensor \( i \), \( \sigma^2_{\beta_i} \) \((i = 1,2,\ldots,n)\) is the elevation errors of the sensor \( i \).

4.2 Scenarios Design and Performance Analysis

In this example, the simulation results are given for the cases of two sensors and three sensors respectively.

Two sensors: Sensor 1: \((5km,0,0)\) Sensor 2: \((-5km,0,0)\)

Three sensors: Sensor 1: \((5km,0,0)\) Sensor 2: \((-5km,0,0)\) Sensor 3: \((0.8,66km,0)\)

The false alarms are generated from a Gaussian distribution with \( \sigma = \text{lnrd} \) centered on the true target position. The number of false alarms is uniformly distributed between 0 and \( \lambda \). The probability of the markovian transition of the system mode is defined as

\[ \Pi = \begin{bmatrix} 0.9 & 0.05 & 0.05 \\ 0.1 & 0.8 & 0.1 \\ 0.1 & 0.1 & 0.8 \end{bmatrix} \]

Two different algorithms were tested using the same trajectory. The target makes two circular turns with rectilinear segments connecting them. The speed modulus is kept constant throughout \((=300m/s)\). The initial position is \((2km,8km,1km)\), and the initial velocity \((150m/s,259.8m/s,0)\). The segments are defined as follows.

1\(^{\text{st}}\) segment. Rectilinear flight until the plane \((6.35km,15.53km,1km)\) \((\text{from } t=0\text{s to } t=30\text{s})\).

2\(^{\text{nd}}\) segment. Circular turn with turn rate \(6^\circ/s\) \((\text{from } t=31\text{s to } t=50\text{s})\).

3\(^{\text{rd}}\) segment. Rectilinear flight until the plane \((14.31km,10.33km,1km)\) \((\text{from } t=51\text{s to } t=70\text{s})\).
4\textsuperscript{th} segment. Circular turn with turn rate 4.8 / s (from $t=71s$ to $t=95s$).

5\textsuperscript{th} segment. Rectilinear flight until the plane (21.26 km, 11.63 km, 1 km) (from $t=96s$ to $t=100s$).

4.2.1 Effect of measurement noise

We now fix the clutter density and the process noise variances, and vary the measurement noise. The clutter density is set to $\lambda = 2$. The process noise variance is set to $Q_{\sigma} = 0.0004$. The simulation results are shown in Fig.1–Fig.2 for two sensors and three sensors respectively. In Fig.1 and Fig.2, all algorithms exhibit a monotonic decrease in performance as $\sigma$ increases. Comparing the RMS error statistics of two sensors and three sensors, it can be found that the performances of all algorithms are a monotonic increase as the number of sensors increases. In Fig.1–Fig.2, it also shows that the IMM-MEFPDAF has the same performance as that of the IMM-PDAF for low measurement noise $\sigma_m = 0.005$ mrad, but significantly superior to the IMM-PDAF for large measurement noise $\sigma_m = 0.01$ or $\sigma_m = 0.06$ when the target emergence maneuver.

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Fig. 1. RMS error statistics for two sensors. RMS Position error (in km) for IMM-MEFPDAF (solid line (-)) , IMM-PDAF (dashed line (--)). Left is for measurement noise ($\sigma_m = 0.0005$ mrad), middle for measurement noise ($\sigma_m = 0.01$ mrad), and right for measurement noise ($\sigma_m = 0.06$ mrad).
Fig. 2. RMS error statistics for three sensors. RMS Position error (in km) for IMM-MEFPDAF (solid line (-)), IMM-PDAF (dashed line (--)). Left is for measurement noise ($\sigma_w = 0.005$ mrad), middle for measurement noise ($\sigma_w = 0.01$ mrad), and right for measurement noise ($\sigma_w = 0.005$ mrad).

4.2.2 Effect of clutter density

We now fix the measurement noise and the process noise variances, and vary the measurement noise. The measurement noise is set to $\sigma_w = 0.01$ mrad. The process noise variance is set to $Q = 0.0004$. The simulation results are shown in Fig.3~Fig.4 for the cases of two sensors and three sensors respectively. In Fig.3 and Fig.4, all algorithms exhibit a monotonic decrease in performance as clutter density $\lambda$ increases. However the degradation in performance is less for the IMM-MEFPDAF than that for the IMM-PDAF. The performance difference between the PDAF and the MEF-PDAF appears to increase with the increment of clutter density. At the same time, it can be found that the IMM-MEFPDAF is significantly superior to the IMM-PDAF when the target emergence maneuvers. Compared the Fig.3 with the Fig.4, it can be found the performance of all algorithms is a monotonic increase as the number of sensors increases.
Fig. 3. RMS error statistics for two sensors. RMS Position error (in km) for IMM-MEFPDAF (solid line (-)), IMM-PDAF (dashed line (--)). Left is for clutter density ($\lambda=0.5$), middle for clutter density ($\lambda=1.5$), and right for clutter density ($\lambda=2.5$).

Fig. 4. RMS error statistics for three sensors. RMS Position error (in km) for IMM-MEFPDAF (solid line (-)), IMM-PDAF (dashed line (--)). Left is for clutter density ($\lambda=0.5$), middle for clutter density ($\lambda=1.5$), and right for clutter density ($\lambda=2.5$).

Finally, the runtime statistics of two algorithms for the case of two sensors are illustrated in Table 1. In each case, the program is run on a computer with a CPU of the Pentium 4 3.0GHZ, 1.0GB RAM, and 50 Monte Carlos runs. As is clearly seen in Table 1, it is shown that the runtime of the proposed algorithms is less than that of the conventional IMM-PDAF. One major reason for this is that in the proposed algorithms if the distance between the measurement and the cluster center exceeded the maximum validate distance, the measurement can be considered as invalidate and eliminated from the validate measurement set; while the measurement was still considered to
be a validate measurement in IMM-PDAF. Moreover, it can be found that the difference of the runtime of two algorithms appears to increase with the increment of clutter density. So the proposed algorithm is much faster than the IMM-PDAF, and can be applied to real time maneuvering target tracking.

<table>
<thead>
<tr>
<th></th>
<th>IMM-PDAF</th>
<th>IMM-MEFPDAF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda = 1$</td>
<td>10.700</td>
<td>9.1400</td>
</tr>
<tr>
<td>$\lambda = 2$</td>
<td>12.1050</td>
<td>10.2350</td>
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<tr>
<td>$\lambda = 3$</td>
<td>14.5350</td>
<td>10.9400</td>
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</tbody>
</table>

**5. Conclusion**

In this paper, we described an efficient and novel approach for maneuvering target tracking based on maximum entropy fuzzy probabilistic data association. Firstly, the derivation of maximum entropy fuzzy clustering to single target tracking is being introduced. Secondly, the integration of interacting multiple models and maximum entropy fuzzy probabilistic data association is described, and applied to the maneuvering target tracking with multiple passive sensors in clutter environment. Simulation results show the performance of this approach is better than the conventional IMM-PDAF approach, and achieve the real-time requirement of maneuvering target tracking. Moreover, the performance of the proposed algorithm did not degrade significantly with an increase in the difficulty of the association problem or the number of clutters.

**Reference**


