Abstract

In this paper, we propose the block-adaptive windows that are used to upgrade the image denoising performance of the doubly local Wiener filtering method and the corresponding algorithm is also used to reduce spatially non-stationary additive white Gaussian noise (SNS-AWGN) in images. Based on the fact that the energy clusters in the detail subimages of an image exhibit direction features varying with spatial locations and oriented subbands, a noisy detail subimage is divided into non-overlapping small-size blocks and the spatial energy correlation function of each block is calculated to determine the principal direction of energy clusters within each block and the corresponding block-adaptive window. The block-adaptive windows are used to improve the estimations of the image's energy distribution in the detail subimages. For noisy images corrupted by SNS-AWGNs, non-uniform noise variances in the pixel domain must be estimated. To do that, we propose the joint neighborhood median absolute deviation (JNMAD) estimator, which makes the denoising algorithm able to be used in the cases of SNS-AWGNs. The experimental results show that the doubly local Wiener filtering method with block-adaptive windows is superior to other wavelet-based methods using two-dimensional separable wavelet transforms in the case of stationary noise and provides satisfactory performance in the cases of SNS-AWGNs.

Keywords: Doubly local Wiener filtering; Block-adaptive window; Spatially non-stationary noise; Joint neighborhood median absolute deviation (JNMAD) estimator

1. Introduction

Images are often corrupted by various types of noises in acquisition and transmission. The primary task of image denoising is to reduce noise while to preserve image information as much as possible. Most of the existing wavelet-based methods are used to reduce spatially stationary additive white Gaussian noises (SS-AWGNs). In that case, estimation of the noise statistics is easy and the key step involves statistical modeling of images and filtering in wavelet domain. The existing models exploit subtly one or more of the following properties: interscale and intrascale dependency, and the distributions of image’s wavelet coefficients. Based on the simple model where the wavelet coefficients of images are assumed to be independent and identical distributed (i.i.d), various thresholding
and shrinkage rules have been established that are not spatially adaptive [1–7]. The spatially adaptive methods exploit more complicated but also more accurate models to capture nonlinear dependency among wavelet coefficients, such as the models of interscale dependency [8–11], the models of intrascale dependency [12–16], and the models of both interscale and intrascale dependency [17–21]. Except for these models, the underlying transforms are also important to denoising performance. It has been shown that the overcomplete bases such as undecimated wavelet transforms [3,14,24], the oriented basis [21], and the curvelet [25], often outperform orthonormal wavelet bases. Additionally, there exist several methods to reduce stationary non-Gaussian or colored Gaussian noise in images [27–31].

Natural images are composed of smooth regions, edges, and textures, and exhibit spatially non-stationary behavior. In the detailed subimages of the wavelet transform, the image’s energy is mainly concentrated in the energy clusters, the aggregations of adjacent wavelet coefficients of large magnitude. Energy clusters are resulted from sharp changes around edges and within textures and carry important information of images. It is important to preserve these energy clusters in the denoising procedure. In [12], image wavelet coefficients are modeled as a realization of a doubly stochastic process. Based on the assumption that wavelet coefficients in a square neighborhood have approximately equal variance, the variance of a wavelet coefficient can be estimated from the wavelet coefficients in its square neighborhood by the approximate ML or MAP estimators. Some improved estimations use the samples selected from the square neighborhood [13,14]. The square neighborhoods are also used in Refs. [17,19,21]. The adaptive merging of different shaped subregions results in a reasonable neighborhood size, which can improve the performance of energy distribution estimation [15]. It should be notable that the square neighborhoods impose an assumption that energy distribution of an image is isotropic in detail subimages. In fact, this is not true. Energy in a detail subimage is mainly distributed along energy clusters. Energy clusters have often directions varying with spatial locations and subbands, which depend on the directions of edges or textures and directional selectivity of the subband filters. Therefore, the shapes of neighborhoods are closely related to the quality of image energy distribution estimations. The elliptic directional neighborhoods matching the directional selectivity of the subband filters are exploited to improve denoising performance [38]. It is notable that the shapes of these neighborhoods are invariant with spatial locations.

In this paper, in order to improve the quality of image energy distribution estimations, we propose the block-adaptive windows. The block-adaptive windows are constructed by the following procedure. First, a noisy subimage is partitioned into non-overlapping small-size blocks. Secondly, the energy correlation function of each block (BECOF) is calculated to capture the local geometric feature of the energy clusters in it. Finally, according to the BECOF of a block, an adaptive window is determined whose shape matches the local feature of the data block. Using the block-adaptive windows, the image energy distribution in detail subimages can be more accurately estimated. Combining the block-adaptive windows and the doubly local Wiener filtering scheme [22,38], a new denoising algorithm is obtained.

Moreover, we extend this algorithm to reduce SNS-AWGNs in images. It is well-known that, in the empirical Wiener filtering in wavelet domain, the image energy distributions and noise variance distributions in the detail subimages need to be estimated. Noise variance distributions in detail subimages may be calculated from the noise variance distribution in the pixel domain by the corresponding energy filter bank. The estimation of the noise variance distribution in the pixel domain becomes the focus of denoising problem when noise is SNS-AWGN. To do that, we propose the joint neighborhood median absolute deviation (JNMAD) estimator, where the single-level undecimated wavelet decomposition of a noisy image is used, and the three neighborhoods in the three oriented subimages are jointly used to increase available samples in the local median absolute deviation (MAD) estimator and the morphological filtering is used to eliminate outliers from image’s edges and textures.

The paper is organized as follows. Section 2 describes the doubly local Wiener filtering method with block-adaptive windows for image denoising. In Section 3, the JNMAD estimator is proposed and its performance is evaluated by numerical experiments. Section 4 compares the performance of the proposed method with those of other state-of-art image denoising methods in the cases of SS-AWGNs, and gives the experimental results in the
cases of SNS-AWGNs. Finally, in Section 5 we conclude our paper.

2. Doubly local Wiener filtering scheme with block-adaptive windows

In this section, the doubly local Wiener filtering scheme [38] is reviewed, where noise variance distributions in detail subimages are assumed to be known. The block-adaptive windows are proposed to estimate the image’s energy distributions in the detail subimages from noisy wavelet coefficients.

2.1. Doubly local Wiener filtering scheme

A noisy observation image \( y(i, j) \) is represented by

\[
y(i, j) = x(i, j) + \varepsilon(i, j), \quad i, j = 1, 2, \ldots, N
\]

where \( x(i, j) \) and \( \varepsilon(i, j) \) are the gray levels of an image and noise sample at the location \((i, j)\), and \( x(i, j) \) and \( \varepsilon(i, j) \) are assumed to be mutually independent. The noise \( \varepsilon(i, j) \) is a realization of a 2-D zero-mean white Gaussian stochastic process whose spatial correlation function (SCF)

\[
r(i, j; l, f) = E[\varepsilon(i, j)\varepsilon(\ell, j)],
\]

\[
= \sigma^2(i, j)\delta(i - \ell)\delta(j - f),
\]

where \( E[\bullet] \) denotes expectation and \( \delta(\bullet) \) is the Kronecker delta. \( \sigma^2(i, j) \) is referred to as the noise variance distribution in the pixel domain. It is assumed to be known in this section and its estimation will be investigated in the next section.

In the wavelet domain, the wavelet coefficients of the noisy image are represented by

\[
w_{k, l}(i, j) = s_{k, l}(i, j) + \varepsilon_{k, l}(i, j),
\]

where the subscripts \( k \) and \( l \) label the scales and oriented subbands, respectively, and \( s_{k, l} \) and \( \varepsilon_{k, l} \) are the wavelet coefficients of the noise-free image and noise, respectively. \( \varepsilon_{k, l}(i, j) \) is zero-mean Gaussian random variable of variance \( \sigma^2_{k, l}(i, j) \).

The noise variance distributions \( \sigma^2_{k, l}(i, j) \) in sub-images can be calculated from \( \sigma^2(i, j) \) by the energy filter bank. Let \([h, g]\) be an orthonormal/biorthogonal filter bank for decomposition. The corresponding equivalent filter bank \( h_l^j(z) \), \( g_l^j(z) \), \( l = 1, 2, \ldots, L \) are represented in z-transform as

\[
H_l^j(z) = H(z)H(z^2)\ldots H(z^{2^{l-1}}), \quad G_l^j(z) = H(z)H(z^2)\ldots H(z^{2^{L-1}})G(z^{2^{L-1}}).
\]

The equivalent filter bank \( h_{k, l}(m, n) \), \( k = 0, 1, 2, 3, l = 1, 2, \ldots, L \) of the corresponding 2-D tensor wavelet are written as

\[
H_{0, l}(z_1, z_2) = H_l^j(z_1)H_l^j(z_2), \quad H_{1, l}(z_1, z_2) = G_l^j(z_1)H_l^j(z_2),
\]

\[
H_{2, l}(z_1, z_2) = H_l^j(z_1)G_l^j(z_2), \quad H_{3, l}(z_1, z_2) = G_l^j(z_1)G_l^j(z_2),
\]

where \( k \) denotes the oriented subband and \( l \) denotes the level of decomposition. The noise variance distributions in subimages are calculated by [36]

\[
\sigma^2_{k, l}(i, j) = \sum_{m} \sum_{n} [h_{k, l}(m, n)]^2 \sigma^2(i - m, j - n).
\]

The filter bank \([h_{k, l}(m, n)]^2\) is referred to as the energy filter bank of the 2-D wavelet system.

The doubly local Wiener filtering is a general framework of the wavelet-based image denoising [22,23,38]. From the first Wiener filtering, a relatively ‘clean’ pilot image is obtained so as to better estimate the image energy distributions in wavelet domain. The second Wiener filtering is used to further improve denoising performance by means of the improved image energy distributions. The flowchart of the doubly local Wiener filtering scheme is shown in Fig. 1, where the DWT, SWT, IDWT and ISWT denote the 2-D separable orthonormal wavelet transform, undecimated wavelet transform, and their inverse transforms, respectively.

Since filtering is performed independently for each subimage, for simplicity we ignore the subscripts of scale and oriented subbands. In the first local Wiener filtering, the image energy distribution is estimated by

\[
\tilde{E}(i, j) = \max \left\{ 0, \frac{1}{\#N_1} \sum_{(p, q) \in N_1} (w_l^1(i + p, j + q) - \sigma^2_l(i + p, j + q)) \right\},
\]

Fig. 1. Flowchart of the doubly local Wiener filtering algorithm.
where \( w_1(i,j) \) is the noisy wavelet coefficients under DWT1/SWT1, \( \sigma_1^2(i,j) \) is the noise variance distribution in the subimage under DWT1/SWT1, and \( N_1 \) and \#\( N_1 \) are a window and its size. The image wavelet coefficients are estimated by the empirical Wiener filtering, that is

\[
\hat{s}_1(i,j) = \frac{\hat{E}(i,j)}{\hat{E}(i,j) + \sigma_1^2(i,j)} w_1(i,j).
\] (8)

A ‘clean’ image is recovered from \( \hat{s}_1(i,j) \) by the inverse transform IDWT1/ISWT1. The recovered image contains much less noise than the original noisy image and is used as a pilot image in the second Wiener filtering. In the second local Wiener filtering, the image energy distributions are re-estimated from the wavelet coefficients \( \hat{s}_2(i,j) \) of the pilot image under the DWT2/SWT2 by

\[
\hat{E}(i,j) = \frac{1}{\#N_2} \sum_{(p,q)\in N_2} \hat{s}_2^2(i+p,j+q),
\] (9)

where \( N_2 \) and \#\( N_2 \) are a window and its size. The image wavelet coefficients are estimated by the empirical Wiener filtering, that is,

\[
\hat{s}_2(i,j) = \frac{\hat{E}(i,j)}{\hat{E}(i,j) + \sigma_2^2(i,j)} w_2(i,j),
\] (10)

where \( w_2(i,j) \) are the wavelet coefficients of the noisy image under the DWT2/SWT2 and \( \sigma_2^2(i,j) \) is the noise variance distributions in the detail subimages of the DWT2/SWT2. The image is recovered from \( \hat{s}_2(i,j) \) by the inverse transform of the DWT2/SWT2. It has been shown that better denoising performance is achieved when the two different wavelet bases are used in the doubly Wiener filtering [38].

Estimations of image energy distributions in (7) and (9) are closely related to the final denoising performance. Performance of the two estimators depends on the sizes and shapes of the windows. For stationary AWGN case, the primary guides on selection of window sizes have been given in [38]. These guides can be summarized as follows. First, for an orthonormal wavelet transform, the window sizes should gradually reduce from the finest scale to the coarsest scale. Secondly, windows of larger size are suitable for the strong noise whereas windows of smaller size for the weak noise. The two guides remain valid in the cases of SNS-AWGNs, though the error formulae of the estimators are complicated. Compared with the sizes of windows, the shapes of windows are more important. Below, we will investigate how to improve image energy distribution estimation using the block-adaptive windows.

2.2. Block-adaptive windows based on block energy correlation function (BECOF)

In [12], the square windows without directivity are used, which results in the fact that estimated energy distribution is badly blurred. In [13–15], the samples selected from a square window is used to improve energy distribution estimation. In [38], the elliptic directional windows matching the directional selectivity of oriented subbands are used to improve image energy distribution estimations. However, the energy distribution estimated using such a fixed window is blurred along the direction of the windows in some regions where the direction of the window mismatches the directions of energy clusters. In Fig. 2, we demonstrate the energy distributions of the three oriented subbands of the test image ‘Barbara’ and their BECOFs. We observe that, in most of data blocks, the energy clusters are distributed along a special direction, which is referred to as the principal direction of the data block. Obviously, for a data block, when the direction of the window matches its principal direction, the blur will be significantly reduced. In fact, the block processing idea has been recently used. In [16], the block covariance matrices of wavelet coefficients are used to improve the performance of the Wiener filtering. Due to the 2-order decorrelation of wavelet transform, the block covariance matrices, 2-order local statistics of the wavelet coefficients, contain less information on energy clusters in data blocks. As a result, the improvement in performance is not obvious. Below, we use the BECOF of a data block, 4-order local statistics of wavelet coefficients, to capture the direction of the energy clusters within it. Then, based on the BECOF, a block-adaptive directional window is determined.

Let \( w_1(i,j), i, j = 0, 1, \ldots, 2^J - 1 \) be noisy wavelet coefficients in a detail subimage and \( \sigma_1^2(i,j) \) be its noise variance distribution under the DWT1/SWT1. Segment the subimage into non-overlapping blocks of size \( 2^j \) by \( 2^j \),

\[
\{ w_1(2^j m + i, 2^j n + j), i, j = 0, 1, \ldots, 2^j - 1 \},
\]

\[
m, n = 0, 1, \ldots, 2^{J-j} - 1,
\]
where \( m \) and \( n \) label the spatial locations of blocks.

For each data block, its BECOF is defined by

\[
\text{BECOF}_{m,n}(p, q) = \sum_{i=0}^{2^j-1} \sum_{j=0}^{2^j-1} z_i^2(2^j m + i, 2^j n + j) \times z_i^2(2^j m + i + p, 2^j n + j + q),
\]

where \(-2^j \leq p, q \leq 2^j\) and \( z_i(i,j) = \max\{0, w_i(i,j) - \sigma_i(i,j)\} \) is a rough estimate of the energy distribution. The BECOFs are 4-order local statistics of wavelet coefficients and thus can capture the intra-dependency, in particular, the principal directions of energy clusters. For example, for the Barbara image corrupted by stationary AWGN of \( \sigma = 10 \), the BECOFs of the three decimated oriented subimages in the second level of the wavelet decomposition are plotted in Fig. 2(e–f), where the size of the subimages is 256 \( \times \) 256 and the block size is 8 \( \times \) 8. It can be seen that the BECOF reflects the principal direction of the energy clusters in each block.

Given the window size \( Q \), we select the shape of the window for each block in terms of its BECOF by

\[
N_1(m,n) = \{(p,q): \text{BECOF}_{m,n}(p,q) \geq \gamma\},
\]

\( \gamma \) is the \( Q \)th largest entry of matrix \( \text{BECOF}_{m,n} \). (12)

Further, the image energy distribution in a subimage for the first Wiener filtering is block-wise estimated by

\[
\hat{E}(i,j) = \max\{0, \frac{1}{\#N_1(m,n)} \times \sum_{(p,q) \in N_1(m,n)} (w_i^2(i+p,j+q) - \sigma_i^2(i+p,j+q))\}
\]

where \( m = \text{floor}(2^{-J_1} i), n = \text{floor}(2^{-J_1} j) \), (13)

where the floor(\( x \)) rounds \( x \) to the nearest integer towards minus infinity. Since the shape of the block-adaptive window matches the principal direction of the energy clusters in the block, the image energy distribution is better estimated. As shown in Fig. 3, we compare the several estimators using the 7 \( \times \) 7 square windows [12], the elliptic window with \( r = 4, a = 1/2 \) [38], and the block-adaptive windows (the window size is 45 and the block size is 8 \( \times \) 8). It can be seen that the best estimate is given by the block-adaptive windows. The square window gives a badly blurred estimate. The energy clusters along other directions are badly blurred when the high elliptic window is used.

In the second Wiener filtering, the BECOFs are estimated from the wavelet coefficients of the pilot image under the DWT2/SWT2 and noise variances are assumed to be zero. The block adaptive windows of smaller size are determined in the similar manner as done in (12) and the image energy distributions are re-estimated in the similar manner as done in (13).
In what follows, we consider the efficient computation of the BECOFs. The formula (11) is of high computational cost. Here, we use the fast Fourier transform (FFT) and the inverse FFT to compute the BECOFs. Every BECOF requires \(2^{2J_1} + 1\) \(2^{2J_1}\) times of multiplications. For a subimage of \(2^{J_1} \times 2^{J_1}\), all the \(2^{2J_2-2J_1}\) BECOFs require \(2^{2J_2} + 1\) \(2^{2J_2}\) times of multiplications.

3. Spatially non-stationary noise and JNMAD estimator

In the above section, the noise variance distribution in the pixel domain is assumed to be known. Because only noisy images are available in image denoising, the noise variance distribution in the pixel domain must be estimated from a noisy image. When noise is spatially stationary zero-mean AWGN (SS-AWGN), the noise statistics are determined by the noise variance \(\sigma^2\), which can be accurately estimated by applying the MAD estimator to the noisy wavelet coefficients in the finest diagonal subimage [26]. Most of the existing wavelet-based algorithms are used to reduce SS-AWGNs in images. In what follows, we consider a slightly complicated noise model, spatially non-stationary additive white Gaussian noise (SNS-AWGN), and the estimation of the variance distributions of SNS-AWGNs.

3.1. Spatially non-stationary additive white Gaussian noise model

A SNS-AWGN \(\varepsilon(i,j)\) has a SCF:

\[
r(i,j; i', j') = E(\varepsilon(i,j)\varepsilon(i', j')) = \sigma^2(i,j)\delta(i - i')\delta(j - j')
\]

Noise variance distribution \(\sigma^2(i,j)\) determines its statistics and noise samples at different spatial locations are mutually independent. SNS-AWGNs belong to a simple subclass of the hetero-scedastic stochastic processes, which have been widely investigated in the statistics [31,32] and are used in speech enhancement and recognition [30,33]. As a common practice, the variance distribution of an SNS-AWGN is assumed to be slowly varying with spatial locations, that is, the noise is approximately stationary in local regions enough small. In image denoising, this is an unavoidable assumption because it is hardly estimated from a noisy image when the noise variance distribution is fast varying with spatial locations.
3.2. Estimation of noise variances using JN MAD estimator

Accurate estimation of the noise variance of a SS-AWGN by the MAD estimator is owing to the two facts: the finest diagonal subimage contains a mass of samples and only a fraction of the samples are outliers that are dominated by edges and textures of an image. In the cases of SNS-AWGNs, these two preconditions no longer hold. The noise variance at each pixel cannot but be estimated from its adjacent pixels, where noise wavelet coefficients have approximately equal variances. As a result, available samples are limited and outliers are also not sparse for the pixels within or around energy clusters. In order to obtain better estimates, we must try to increase available samples and to reduce outliers. Along this line, we propose the JN MAD estimator, where the three neighborhoods in the three oriented subimages are jointly used to increase available samples and the thresholding and the morphological filtering [32] are used to remove outliers in these three neighborhoods.

In what follows, we observe noise variances in the four subimages of a single-level undecimated wavelet decomposition of a noisy image. Let a 2-D orthogonal or biorthogonal filter bank \( h_k(m,n), k = 0, 1, 2, 3 \) to be used, the four subimages (LL, HL, LH, and HH subbands) are restricted by the local stationarity of noise. We use the local MAD estimator in (17), the available samples are re-estimated using

\[
\hat{\sigma}_1(i,j) = 1.4826 \text{MAD} \left[ |w_{3,1}(i + p, j + q)|, (p, q) \in \Omega_n \right],
\]

(16)

is used, where \( \Omega_n \) is a neighborhood of \((0,0)\) in \( Z \times Z \times (Z \text{ denotes the integer set}) \). For the pixels within or around energy clusters, their neighborhoods contain relatively more outliers, which make the estimates at these pixels to be biased upward. Next, we remove the samples of magnitudes more than 3 \( \hat{\sigma}_1(i,j) \) from the HH subimage. Noise variances are re-estimated using

\[
\hat{\sigma}_2(i,j) = 1.4826 \text{MAD} \left[ |w_{3,1}(i + p, j + q)| \leq 3\hat{\sigma}_1(i,j), (p, q) \in \Omega_n \right].
\]

(17)

The MAD estimator is asymptotically unbiased for Gaussian random variables. If the random variables are i.i.d, the bias of the MAD estimator is very small when the number of samples is more than 50. The variance of the MAD estimator is proportional to the variance of the random variables and inversely proportional to the square root of the sample number [37]. For the local MAD estimator in (17), the available samples are restricted by the local stationarity of noise. We use the adjacent wavelet coefficients in the three detail subimages jointly to increase available samples. Before estimation, the thresholding and morphological filtering are used to remove outliers in the three detail subimages. Define \( \Lambda_k = \{(i,j): |w_{k,1}(i,j)| > 3\hat{\sigma}_2(i,j)\}, \ k = 1, 2, 3 \), where \( \hat{\sigma}_2(i,j) \) is the standard deviation estimated by (17). The dilation operator [34] is used to expand the sets \( \Lambda_k \). As a result, we obtain

\[
\Lambda_k^* = \Lambda_k \oplus S = \{(p, q): \Lambda_k \cap [(p, q) + S] \neq \emptyset\},
\]

where \( S \) is the structuring element and \( \Phi \) represents the empty set. The size of \( S \) is related to the support region of the wavelet filters and longer filters require the structuring element of larger size because it spreads a singular point of an image into a larger region in the wavelet domain. Here, we use the short wavelet filters in the single-level undecimated decomposition and thus \( S \) is taken as a circular set \( \{(p, q): p^2 + q^2 \leq 2^2\} \). The set \( \Lambda_k^* \) contain most of the
outliers in the detail subimages. The noisy wavelet coefficients outside \( A_k^c \) are used to re-estimated noise variances by
\[
\hat{\sigma}_j(i, j) = 1.4836 \text{MAD}(|w_k,i(i + p, j + q)|): \quad (p, q) \in \Omega_n \text{ and } (i + p, j + q) \notin A_k^c, k = 1, 2, 3
\]
(18)

We refer to (18) as the JNMAD estimator.

Once the noise variance distribution in the pixel domain is estimated, we can use it to replace the true noise variance distribution in Fig. 1. In this way, the doubly local Wiener filtering algorithm can be applied to the cases of SNS-AWGNs.

The size of the neighborhood \( \Omega_n \) and the wavelet filters used in the single-level undecimated decomposition are related to the performance of the JNMA estimator. Here, we use the circular neighborhoods of the form
\[
\Omega_n(r) = \{(p, q) : p^2 + q^2 \leq r^2\},
\]
(19)
where \( r \) is its radius. It is easy to observe that, the samples in a neighborhood too large have quite different variances and the estimator has a large bias, while the available samples in a neighborhood too small are few and the estimator has a large variance. The selection of the size of neighborhoods substantially depends on the local stationary of noise. The radius \( r \) of \( \Omega_n(r) \) can be empirically selected in terms of the local stationary of noise.

In what follows, we consider the wavelet filters in the single-level decomposition. An image is typically composed of smooth regions, edges, and textures. For the local MAD and JNMA estimators, it is desirable for the wavelet filters to have the following characteristics: first, high vanishing moments such that the wavelet coefficients of the image in smooth regions approximate zero and the noisy wavelet coefficients in these regions are dominated by noise; secondly, short support such that edges are expanded less in detail subimages and consequently the number of outliers are reduced. The B-spline filters meet the two requirements. The \( m \)-order cardinal B-spline filters can be written in \( z \)-transforms as [35]
\[
\begin{align*}
N_m^L(z) &= c_m(1 + z)^m, \\
N_m^H(z) &= c_m(1 - z)^m, \\
c_m &= \left( \sum_{k=0}^{m} \frac{m!}{k!(m-k)!} \right)^{1/2},
\end{align*}
\]
(20)
where \( c_m \) is a constant to normalize the filters.

3.3. Numerical experiments for JNMAD estimator

In what follows, we evaluate the performance of the JNMAD estimator by numerical experiments. For convenience, the variances of a test SNS-AWGN are written as
\[
\sigma^2(i, j) = \hat{\sigma}^2 \Theta^2(i, j),
\]
(21)
\[i, j = 1, 2, \ldots, I \text{ and } \sum_{ij} \Theta^2(i, j) = I^2,
\]
where \( \sigma^2 \) and \( \Theta^2(i, j) \) are the average variance and the normalized distribution, respectively. Given them, a SNS-AWGN is generated by
\[
\epsilon(i, j) = \sigma \Theta(i, j) \text{randn}(i, j), \quad i, j = 1, 2, \ldots, I,
\]
(22)
where the \( \text{randn}(i, j) \) generates a Gaussian random number of zero-mean and unit variance. In numerical experiments, all noise samples have the same normalized distribution as shown in Fig. 4. The estimators are evaluated by the relative error (RE)
\[
\text{RE} = \frac{\sum_{ij}(\sigma(i, j) - \hat{\sigma}(i, j))}{\sum_{ij}\sigma(i, j)},
\]
(23)
where \( \hat{\sigma}(i, j) \) are the standard deviations estimated by the local MAD or JNMA estimators.

In numerical experiments, the test images includes the five commonly used 512 \( \times \) 512 8-bits gray images, ‘Lena’, ‘Barbara’, ‘Peppers’, ‘Goldhill’, and ‘Boats’. When \( \hat{\sigma} = 15 \) and 20, the REs (the average values of 10 independent tests) are listed in Table 1, where 4-order cardinal B-spline filter bank is used and the neighborhoods \( \Omega_n(20) \) and \( \Omega_n(15) \) are used for the local MAD and JNMA estima-

![Fig. 4. Normalized noise variance distribution for generation of SNS-AWGN sample.](image.png)
tors. The JNMAD estimator yields better performance than the local MAD estimator. The JNMAD estimator yields relatively small improvement for the test images ‘Barbara’ and ‘Goldhill’ with abundant textures, which is because the fact that their detail subimages contain too many outliers from textures. For a noisy ‘Lena’ image with $\bar{\sigma} = 20$, the standard deviation distributions estimated by the local MAD estimator and the JNMAD estimator are shown in Fig. 5(a) and (b), respectively. It can be seen that the result in Fig. 5(b) is closer to the true distribution in Fig. 4.

In Table 2, we give the relative errors of the JNMAD estimator using different wavelet bases for $\bar{\sigma} = 10$. It can be seen that the 3- and 4-order B-spline wavelet bases are better than the other filters, which accords with our previous analysis. In below, the 4-order B-spline filters are always used in the JNMAD estimator.

4. Experimental results

4.1. Denoising performance in SS-AWGN case

In order to demonstrate the effectiveness of the block-adaptive windows, we compare our method with some of the currently available methods on the two $512 \times 512$ 8-bits grayscale test images, ‘Lena’ and ‘Barbara’ in the cases of SS-AWGNs. In the cases of SS-AWGNs, the noise variance is estimated by the MAD estimator.

For orthonormal wavelet transform case, the wavelet base ‘db4’ of length 8 and ‘sym8’ with eight vanishing moments are used to the first and second Wiener filtering, respectively. The five-level decomposition with boundary symmetric-padding is used. Each subimage is divided into $8 \times 8$ blocks.
for the first Wiener filtering are \{51,41,31,21,11\} + \sigma and the window sizes for the second Wiener filtering are \{25,20,15,10,5\}. The output PSNRs are listed in Table 3, where each PSNR is the average of the twenty independent tests. Among the methods listed in Table 3, the doubly local Wiener filtering method with block-adaptive windows gives the best results. This improvement in performance is owing to the doubly local Wiener filtering scheme and the block-adaptive windows. Comparing with the algorithms in [12,15,17] that use single Wiener filtering, the algorithm [38] using doubly Wiener filtering obtains about 0.3 dB PSNR improvement for the ‘Lena’ image and about 0.5 dB PSNR improvement for the ‘Barbara’ image. Comparing with the algorithm [38], the proposed algorithm using block-adaptive windows obtain about 0.2 dB PSNR improvement for the two test images. To compare visually denoising effect, we present a local region of the denoising images in Fig. 6.

For the undecimated wavelet transform case, the same wavelet bases as well as five-level undecimated decompositions with boundary symmetric-padding are used. The block size is 16 × 16. The window size for first Wiener filtering is 131 + 2\sigma for all levels and the window size for the second Wiener filtering is 71 for all levels. The output PSNRs of our algorithm and some state-of-the-art algorithms are listed in Table 4. It can be seen that, our algorithm gives better denoising effect. In particular, for the test image ‘Barbara’ with abundant edges and textures, the average improvement of PSNR is close to 0.4 dB over the D-T CWT algorithm in [17]. Comparing with the DFB-GSM algorithm using directional filter banks and a more complicated statistical model, our algorithm has about 0.2 dB degradation of the output PSNRs. For the DFB-GSM algorithm, the computational cost mainly comes from the pyramid transform of the directional filter bank and the estimation procedure in the transform domain. According to the conclusion in [21], the computational cost of an oriented subband of size 2^j \times 2^j scales as 2^{2j} and the computational cost of the estimation procedure scales NKS_\sigma 2^{2j} (in [21], N = 10, S_\sigma = 13, K = 8 is the number of orientations, i.e.,NKS_\sigma = 1040). The computational cost of the proposed method mainly comes from the undecimated wavelet transform and calculation of the BECOFs. The computational cost of a subimage
of size $2^J \times 2^J$ scales $L^2 2^{2J}$, where $L$ is the length of the wavelet filter and the computational cost of all the BECOFs of a subimage scales $(2J_1^2 + 1)2^{2J}$ (where $2^J_1 \times 2^J_1$ is the size of data block (in our algorithm, $J_1 = 4$). It should be noticed that the computational complexity of the DFB is $O(2^{2J})$, because the DFB Transform is implemented by the FFT and the inverse FFT in the frequency domain. The computational complexity of the undecimated wavelet transform (the depth of the decomposition is fixed) is $O(2^{2J})$ when $J$ tends to infinity. Moreover, the computational cost of the BECOFs is much smaller than that of the estimation procedure in the DFB-GSM algorithm. As a result, our algorithm has low computational cost when the size $2^J \times 2^J$ of images is large.

Additionally, the experimental results show that the denoising performance is robust to variation of the window sizes in a wide range for undecimated wavelet transforms. For a noisy ‘Barbara’ image with $\sigma = 15$, we vary the window sizes for the first Wiener filtering from 131 to 281, and for the second Wiener filtering from 31 to 81, the difference of the maximal and minimal PSNRs does not exceed 0.05 dB.

4.2. Denoising performance in SNS-AWGN case

In the cases of SNS-AWGNs, the performance of our algorithm depends on the estimations of the noise variance distribution and image energy distributions. The experimental results in the cases of SS-AWGNs have shown that the block-adaptive windows improve the accuracy of image energy distribution estimations and denoising performance. In what follows, we intend to verify the effectiveness
of the JNMAD estimator. First, we simulate four different types of SNS-AWGNs and their normalized variance distributions are plotted in Fig. 7(a)–(d). In experiments, noise variance distributions in the pixel domain are estimated by the JNMAD estimator with the neighborhood $\Omega_n$ (20) in (16) and (17) and the neighborhood $\Omega_n$ (15) in (18). In the doubly local Wiener filtering, the five-level undecimated wavelet decompositions are used, the wavelet bases ‘db4’ and ‘sym8’ are used, and the block size is $16 \times 16$. From the finest level to the coarsest level, the window sizes for the first Wiener filtering are $\{201,221,241,261,271\}$ and the window sizes for the second Wiener filtering are $\{51,61,71,81,91\}$. For the test image ‘Lena’ and ‘Barbara’, the output PSNRs are listed in Table 5. In order to verify the effectiveness of the JNMAD estimator, we also made the experiments where the true noise variance distributions are used. We find that, the JNMAD estimator only results in about 0.3 dB degradation in output PSNRs comparing with the algorithm using true noise variance distributions.

In Figs. 8 and 9, we demonstrate the denoising results using the true noise variance distribution and using the estimate by the JNMAD estimator. It can be seen that for the ‘Lena’ image with few textures, usage of the JNMAD estimator only yields small degradation in visual and PSNR. For the ‘Barbara’ image with abundant textures, usage of the JNMAD estimator suffers from relatively large degradation in PSNR. On the whole, these show that the algorithm using the JNMAD estimator is acceptable in performance. Additionally, the experiments below show that the JNMAD estimator is necessary in the cases of SNS-AWGNs. When the conventional MAD estimator is used where the SNS-AWGN is processed as the SS-AWGN or the local MAD estimator for SNS-AWGNs is used, denoising performance suffer from degradations with different extents. In Table 6, the output PSNRs of the algorithm using the MAD, local MAD, and the JNMAD estimators are listed for type-II SNS-

| Table 5 | Output PSNRs for our algorithms to use to the four SNS-AWGNs |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Test images     | Lena            | Barbara         | Lena            | Barbara         | Lena            | Barbara         | Lena            | Barbara         |
| Average noise level ($\sigma$) | 10 20 30 | 10 20 30 | 10 20 30 | 10 20 30 | 10 20 30 | 10 20 30 | 10 20 30 | 10 20 30 |
| Type-I SS-AWGN  | 35.35 32.27 30.30 33.37 29.85 27.63 | 35.35 32.27 30.30 33.37 29.85 27.63 | 35.35 32.27 30.30 33.37 29.85 27.63 | 35.35 32.27 30.30 33.37 29.85 27.63 | 35.35 32.27 30.30 33.37 29.85 27.63 | 35.35 32.27 30.30 33.37 29.85 27.63 | 35.35 32.27 30.30 33.37 29.85 27.63 | 35.35 32.27 30.30 33.37 29.85 27.63 |
| Type-II SS-AWGN  | 35.31 32.13 30.16 33.51 29.81 27.62 | 35.31 32.13 30.16 33.51 29.81 27.62 | 35.31 32.13 30.16 33.51 29.81 27.62 | 35.31 32.13 30.16 33.51 29.81 27.62 | 35.31 32.13 30.16 33.51 29.81 27.62 | 35.31 32.13 30.16 33.51 29.81 27.62 | 35.31 32.13 30.16 33.51 29.81 27.62 | 35.31 32.13 30.16 33.51 29.81 27.62 |
| Type-III SS-AWGN | 35.28 32.15 30.28 33.53 29.91 27.71 | 35.28 32.15 30.28 33.53 29.91 27.71 | 35.28 32.15 30.28 33.53 29.91 27.71 | 35.28 32.15 30.28 33.53 29.91 27.71 | 35.28 32.15 30.28 33.53 29.91 27.71 | 35.28 32.15 30.28 33.53 29.91 27.71 | 35.28 32.15 30.28 33.53 29.91 27.71 | 35.28 32.15 30.28 33.53 29.91 27.71 |
| Type-IV SS-AWGN  | 35.47 32.47 30.64 33.57 29.88 27.71 | 35.47 32.47 30.64 33.57 29.88 27.71 | 35.47 32.47 30.64 33.57 29.88 27.71 | 35.47 32.47 30.64 33.57 29.88 27.71 | 35.47 32.47 30.64 33.57 29.88 27.71 | 35.47 32.47 30.64 33.57 29.88 27.71 | 35.47 32.47 30.64 33.57 29.88 27.71 | 35.47 32.47 30.64 33.57 29.88 27.71 |

Fig. 8. Comparison of the proposed algorithm using the true noise variance distribution and the estimate by the JNMAD estimator: (a) noise-free ‘Lena’ image; (b) noisy image (Type-III SNS-AWGN with $\sigma = 20$); (c) denoising result using the true noise variance distribution (output PSNR = 32.36 dB); and (d) denoising result using the estimate by the JNMAD estimator (output PSNR = 32.23 dB).

Fig. 9. Comparison of the proposed algorithm using the true noise variance distribution and the estimate by the JNMAD estimator: (a) noise-free ‘Barbara’ image; (b) noisy image (Type-II SNS-AWGN with $\sigma = 15$); (c) Denoising result using the true noise variance distribution (output PSNR = 31.75 dB); and (d) Denoising result using the estimate by the JNMAD estimator (output PSNR = 31.40 dB).
AWGNs. It brings severe degradation in performance to use the conventional MAD estimator in the cases of SNS-AWGNs. The algorithm using the JNMAD estimator outperforms one using the local MAD estimator.

5. Conclusions

The doubly local Wiener filtering is an efficient and fast approach for image denoising. In the cases of SS-AWGNs, the denoising performance highly depends on the quality of estimations of image energy distributions. In this paper, we proposed the block-adaptive windows, by which the quality of estimations of image energy distributions is significantly improved. The doubly local Wiener filtering method with block-adaptive windows obtains better denoising performance. Moreover, we extend the algorithm to the cases of SNS-AWGNs. The JNMAD estimator is proposed to estimate the noise variance distribution in the pixel domain from the three detail subimages of single-level undecimated wavelet decomposition. The experimental results show that the algorithm using the JNMAD estimator effectively reduces SNS-AWGN in images.

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References


