HFSWR based on synthetic impulse and aperture processing

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Abstract: A novel high-frequency surface wave radar (HFSWR) has been proposed, which is based on the principles of synthetic impulse and aperture processing. It uses multiple omnidirectional transmit antennae to simultaneously transmit a set of orthogonal waveforms, and the echoed signals are received and processed by one or multiple reception arrays. Although the transmit beam pattern is omnidirectional, by proper processing, the received signal multiple equivalent directional transmit beam patterns can be simultaneously formed. The signal processing scheme to obtain equivalent directional transmit beam pattern is investigated in detail. Considerations for system parameter selection to achieve an overall best system performance are proposed and discussed. In particular, for the novel HFSWR, target range and angle estimates are coupled together due to the orthogonality of the transmitted waveforms. A necessary and sufficient condition on the transmit antenna array geometry and transmit frequencies, which ensures that target range and angle estimates are uncoupled, is presented.

1 Introduction

High-frequency surface wave radar (HFSWR) operates in HF band (3–30 MHz). In HF band, the electromagnetic waves can propagate by following the curvature of the earth along the air–water interface. Owing to the very low propagation losses of the highly conductive ocean surface, HF radiation can easily propagate beyond the line-of-sight. Therefore, HFSWR can provide over the horizon detection of targets on large oceanic area, which is widely used in both military surveillance and civilian applications [1–4]. As the wavelength of HFSWR is generally of 10–100 m, which approximates to the physical size of ships and large aircrafts, the radar cross-section (RCS) of these targets is more dependent on their gross dimension than that of shape details. Therefore HFSWR has the potential to detect stealth targets to which RCS reduction technique has been used by proper shape design. Besides, the observed Doppler spectrum of sea echoes with a HFSWR has peculiar and unique dominant peaks, which are generated by the Bragg scatter from ocean waves with wavelength exactly half the radar wavelength [5, 6]. Therefore sea surface currents as well as other sea-state information can be extracted from the sea echoes’ spectrum, because sea surface currents always cause the first-order Bragg resonant frequencies to be shifted by a small amount from their predicted positions, and the second-order sea echoes contain much information about wave properties. In conclusion, HFSWR offers the capability of inexpensive surveillance of a large area as well as monitoring of exclusive economic zone, with the ability to track targets and protect environments continuously and in all weathers.

The synthetic impulse and aperture radar (SIAR) is a 4-D radar, namely, it can measure target range, azimuth angle, elevation angle and Doppler frequency. It uses multiple omnidirectional transmit antennae to simultaneously radiate a set of orthogonal waveforms, and a reception array to receive target-echoed signals of all the transmit antennae [7–9]. As each transmit antenna is omnidirectional, and their transmitted waveforms are orthogonal to each other, the overall transmit beam pattern is omnidirectional. However, multiple equivalent directional transmit beam patterns can be obtained by performing digital time-space beamforming on received signals, which is also called as synthetic impulse and aperture processing (SIAP). This gives rise to increased dwell time and improved target detection performance of SIAR compared with the conventional radar. Besides, SIAR has many other advantages, such as, low probability of interception, flexible system construction, high reliability and so on.

By using pulse compression technique, frequency modulated interrupted continuous waveform (FMICW) can achieve good range resolution and large maximum range with a relatively high duty cycle [10, 11]. It can also provide good isolation between the transmitter and the receiver, thus allowing a relative higher power monostatic operation of HFSWR. Therefore FMICW is an ideal waveform for HFSWR.

The traditional HFSWR generally works in a monostatic mode. By applying the SIAP to HFSWR, we propose a novel HFSWR, which uses multiple omnidirectional transmit antennae to transmit FMICWs with different carrier frequencies, namely, the transmitted signals are orthogonal to each other. The target echoed signal is received and processed by one or multiple reception antennae. In theory, the directional transmit antenna array beam pattern can be formed by proper processing of the received signals. In addition, the external interference can be suppressed via adaptive beamforming based on a reception antenna array. The system construction of the proposed HFSWR is flexible, it can operate in monostatic, bistatic or multistatic mode, and the reception antenna can be installed on...
ground, ground vehicles or other moving platforms. In addition to the advantages of the conventional HFSWR and SIAR, as mentioned earlier, the HFSWR proposed in this paper also has the advantages of good electronic counter-countermeasures (ECCM) performance, high agility and manoeuvrability and good survivability. In this paper, the signal processing scheme and the system parameters selection issues of the proposed HFSWR are concerned in detail.

2 System overview

2.1 System description

The HFSWR proposed in this paper uses multiple transmit antennae to simultaneously radiate a set of orthogonal waveforms, respectively, as mentioned earlier, the radiation pattern is omnidirectional, target range, azimuth angle and Doppler can be estimated from the target echoed signal via particular signal processing algorithm. As the transmit pattern is omnidirectional, the large transmit array can be established along the coast, and the small reception array can be installed on the ship or the ground vehicle. In addition, multiple reception arrays can be employed to construct a multistatic radar system with the transmit antennae serving as illuminator. Furthermore, the system may be operated as passive coherent location system, for example, utilises a single transmit antenna as illuminator and a reception antenna as a passive receiver.

For each reception antenna, the received signal is amplified by low-noise amplifier, coherent demodulated and sampled by A/D converter to generate high-quality I, Q components. The complex signal is then band-pass filtered digitally to generate multichannel signals (each channel corresponds to a transmit antenna). The signal of each channel is range transformed separately, and the multiple channel range transformed signals are used as the input of time-space beamforming. At this stage, the system has determined the ranges and azimuths of a number of targets. In order to determine the Doppler of each target, it is necessary to further process the data by coherent integration, which will be described in detail in Section 3.

2.2 Characteristics

With above system structure, the proposed HFSWR has several merits.

(i) The fact that the reception is passive makes it far less vulnerable to electronic counter measures.

(ii) Operation in HF band makes it a good means to detect stealth targets.

(iii) Simultaneous multiple equivalent transmit beams can be formed by time-space beamforming on the received signal, which enables it to increase the dwell time for long-time coherent integration, thus improving the detection performance for weak targets.

(iv) Space synchronisation can be automatically achieved by time-space beamforming at reception, thus the system control complexity can be decreased.

(v) Reception array on moving platform provides it with high agility, manoeuvrability and good survivability.

(vi) The target location information estimated by multiple reception arrays can be combined together to provide enhanced target location estimation.

(vii) Extended surveillance area and beyond the line-of-sight target detectability.

(viii) The potential for environmental monitoring.

3 Signal processing scheme

The signal processing scheme at the receiving side plays an important role in the system described in this paper. The targets of interest to an HFSWR are low-flying aircrafts, missiles and ships. So, we only consider target azimuth angle in this paper. Assuming a transmit antenna array of N isotropic elements, and FMICWs with different carrier frequencies are employed as transmit waveforms. As shown in Fig. 1, the location of the nth transmit antenna is given by the position vector \( r_n = [r_n \cos \phi_n, \sin \phi_n]^T \). T denotes transpose operator, \( |r_n| \) is the distance between the nth transmit antenna and the origin of the Cartesian coordinate system, which is selected as reference point and \( \phi_n \) is the azimuth angle of the nth transmit antenna measured counterclockwise from the X-axis.

3.1 Transmit signal

The system utilises an antenna array to transmit orthogonal waveforms, the transmit signal of the nth transmit antenna is

\[
s_n(t) = g(t) e^{j2\pi (f_0 + \Delta f_n) t - (\mu/2)t^2} \quad n = 1, \ldots, N
\]

where \( g(t) \) is the gate signal which controls on and off of the FMICW signal to produce FMICW waveform, \( f_0 \) is the radar carrier frequency, \( \Delta f_n \) is the frequency deviation of the nth transmit antenna relative to \( f_0 \), \( \mu \) is the frequency sweep rate, \( N \) is the number of transmit antenna.

3.2 Multi-channel signals separation

For simplicity, we suppose the HFSWR discussed in this paper operating in monostatic mode and only a single isotropic reception antenna located at the origin of the Cartesian coordinate system. We also suppose there is only one target moving with constant radial velocity. The results for this case can be extended to the cases of multiple targets at various distances and radial velocities and the HFSWR operating in bistatic or multistatic mode. Suppose the target is initially located at distance \( R_0 \) and is moving with constant radial velocity \( v \), the reflected signal from this target will essentially be a delayed and Doppler shifted version of the original transmit signal. Without loss of generality, the signal decay factors corresponding to target RCS and propagation distance of each transmitted signal are assumed equal to each other, and they are omitted in the following deduction. As the transmit signals are orthogonal to each other, the signal received by the reception antenna is vector sum of the reflected signals that are transmitted by the transmit antenna array and reflected by the target. Hence the received signal is

\[
r(t) = \sum_{n=1}^{N} g(t - \tau_n) e^{j2\pi (f_0 + \Delta f_n) (t - \tau_n) - (\mu/2)(t - \tau_n)^2} \quad (2)
\]

![Fig. 1 Transmit antenna array geometry on two dimensional X-Y plane](image-url)
where \( f_t = f_0 + \Delta f_t \), \( \tau_n \) is the delay time introduced by the target distance, target motion and the distance between the \( n \)th transmit antenna and reference point, which is given by

\[
\tau_n = \tau_0 - \Delta \tau_n - \frac{2\nu t'}{c} \quad n = 1, \ldots, N
\]

where \( t' = mT_r + t \), \( 0 \leq t < T_r \), \( T_r \) is the frequency sweep repetition period, \( m \) is the transmitted sweep number index, \( c \) is the velocity of the light, \( \tau_0 = 2R_0/c \), \( \Delta \tau_n = r_nu/c \), \( u = (\cos \theta, \sin \theta)^T \) is the target unit direction vector as shown in Fig. 1.

The received signal is demodulated by multiplying with an un gated version of the transmit signal \( e^{j2\pi f_t(t-t')/c} \), and then sampled by an A/D converter. In order to separate the reflected signals with different carrier frequencies, with each carrier frequency corresponding to different transmit antenna, the sampled signal is passed through multiple channels, and multiplied by \( e^{-j2\pi n_2u/c} \) in each channel separately, and then low-pass filtered individually, as illustrated in Fig. 2. By neglecting the gate signal, the output signal of the \( n \)th channel is given by

\[
r_n(t) = e^{j2\pi \mu t - j2\pi \mu \nu t/c} \quad n = 1, \ldots, N
\]

As \( \tau_0 \gg \Delta \tau_n, \tau_0 \gg 2\nu t'/c \), the instantaneous frequency of the above signal is approximately related to target delay time \( \tau_n \) by \( f_t = \mu \tau_n \).

### 3.3 Range transform

As shown in (4), the range of the specific targets can be estimated from the frequency of demodulated and separated signals. This can be done by performing the fast Fourier transform (FFT) of a single sweep, which is also known as range transform. As demonstrated in Fig. 2, the range transformed signal of the range cell \( R_0 \) at the \( m \)th repetition interval is given as

\[
r_n(R_0, mT_r) = Ae^{j2\pi f_{cT_r}}e^{-j2\pi f_n(t_0-\Delta t)} \quad n = 1, \ldots, N; \quad m = 0, \ldots, M - 1
\]

where \( A \) is the signal amplitude, \( M \) is the number of pulses used for coherent integration, and \( MT_r \) is the coherent integration time. The target velocity is involved in the first exponential term in (5). When certain limitation (11) is satisfied, the Doppler frequency corresponding to radar carrier frequency \( f_0 \), which is denoted by \( f_d \), is equal to \( 2\nu f_0/c \) approximately. Consequently, the range transformed signal in (5) can be written as

\[
r_n(R_0, mT_r) = Ae^{j2\pi f_{cT_r}mT_r}e^{-j2\pi f_n(t_0-\Delta t)} \quad n = 1, \ldots, N; \quad m = 0, \ldots, M - 1
\]

Let \( r_n(R_0, mT_r) = [r_1(R_0, mT_r), r_2(R_0, mT_r), \ldots, r_N(R_0, mT_r)]^T \), denote the multichannel signals after range transform, then we have

\[
r(R_0, mT_r) = s(mT_r)a(R_0, \theta) \quad m = 1, \ldots, M
\]

where \( s(mT_r) = Ae^{j2\pi f_{cT_r}mT_r} \) can be viewed as the complex envelope of the range transformed signal, \( a(R_0, \theta) = [e^{-j2\pi f_0(t_0-\Delta t)}, e^{-j2\pi f_2(t_0-\Delta t)}, \ldots, e^{-j2\pi f_n(t_0-\Delta t)}]^T \) is the equivalent steering vector of the system. Unlike the steering vector of the conventional array, which is a function of target azimuth angle, the steering vector here is a function of both target range and target azimuth angle.

The effect of noise is neglected in the above analysis. At the transmit antenna, the signal-to-noise ratio (SNR) is so high that the effect of transmitter noise can be neglected. At HF band, the external noise due to atmospherics, cosmic noise and man-made noise can be significantly greater than the internal receiver noise generated within the receiver itself. Consequently in a HFSWR, the receiver sensitivity is usually determined by the external noise. Therefore noise effect must be considered at the reception antenna. So, the range transformed multichannel signals may be modified as

\[
r(R_0, mT_r) = s(mT_r)a(R_0, \theta) + n \quad m = 1, \ldots, M
\]

where \( n \) is the noise vector with covariance of \( \sigma^2 I \).

### 3.4 Time-space beamforming

By far, we only use the receiver output multichannel signals independently to determine target range (with range resolution of \( c/2\nu T_r \) for each channel, the overall range resolution of the system will be discussed in detail in Section 4.3) The range transformed multichannel signals can be further processed to improve target range estimation accuracy and to obtain the estimate of target azimuth angle. As shown in Fig. 2, the range transformed multichannel signals, which is given by (8), is further processed by SIAP which is also called time-space beamforming or synthesis processing in this paper. The time-space beamforming weight vector is given by

\[
w(R, \theta) = [e^{-j2\pi f_0(t_0-\Delta t)}, e^{-j2\pi f_2(t_0-\Delta t)}, \ldots, e^{-j2\pi f_n(t_0-\Delta t)}]^T
\]

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**Fig. 2 Signal processing diagram**

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where \( r = 2R/c, \Delta r = [r^2, (\cos \theta, \sin \theta)]/c \), for a certain range cell \( R_0, R \in [R_0 - (\Delta R/2), R_0 + (\Delta R/2)] \), where \( \Delta R = (c/\mu T_s) \) is the range resolution determined by the bandwidth of each transmitted FMICW. Obviously, the output of time-space beamforming is a function of target bandwidth of each transmitted FMICW. The amplitude–range–azimuth (ARA) surface therefore for each range cell, the synthesised output is given by

\[
y(R, \theta, mT_s) = w^b(R, \theta) \cdot r(R_0, mT_s), \quad m = 1, \ldots, M
\]

where \( H \) denotes complex conjugate. Target range with improved accuracy and azimuth angle can be simultaneously determined by examining the ARA surface.

It should be noted that the above synthesis processing can be accomplished on the basis of single omnidirectional receiver. The spatial gain of the received signal equals to the equivalent (or synthesised) transmitted beam gain. In order to further improve the spatial gain of the system, one can adopt multiple receiver antennae, the time-space beamforming output signals of each receiver antenna can be coherent integrated via beamforming, and thus the detection performance can be further improved. In addition, the external interference can be suppressed via adaptive beam-forming if multiple receiver antennae is adopted.

### 3.5 Coherent integration

For weak target or long distance target, the power of its echoed signal after the above processing is generally not strong enough compared with noise power for reliable detection, the SNR should be improved by coherent integrating multiple echoed signals. The coherent integration over a number of sweeps, which is also known as Doppler transform, can be implemented by FFT. This transform gives a good first-order estimate of target radial velocity.

In real-time implementation, overall computation complexity can be greatly reduced by using some efficient algorithms [11, 12] to implement range and Doppler transform.

For the convenience of state, we perform synthesis processing followed by coherent integration as illustrated in Fig. 2, however, their orders are actually interchangeable. To put the coherent integration before the synthesis processing may be helpful in reducing the computational load for target parameters estimation especially under moderate or low SNR circumstances.

### 4 System parameter selection considerations

In order to achieve an overall best system performance, system parameters should be deliberately selected. Some system parameter selection principles, such as frequency sweep repetition period, on and off time of gating sequence and so on are familiar to radar researchers. Thus, in this section, we will mainly discuss the selection of system parameters with respect to the proposed particular radar regime, including operating frequency and transmit array geometry, which play an important role in the proposed system and affects system performance in many respects.

#### 4.1 Transmit waveform orthogonality

In order to keep the orthogonality of the transmit waveforms, transmit frequencies can be selected as a set of orthogonal basis. For example, \( f_0 = f_0 + c_n \Delta f \), \( \Delta f \) is frequency separation, \( c_n \) is the transmit frequency code of the \( n \)th transmit antenna, \( c_n \in \{0, N - 1\}, c_i \neq c_j (i \neq j) \).

#### 4.2 Avoiding Doppler spread

In (6), we have used a constant to represent the Doppler frequency corresponding to different frequency \( f_s \), approximately. In fact, as each transmit antenna has different carrier frequency, the Doppler frequency of target corresponding to each transmit antenna is different too. If the bandwidth of the Doppler filter is too narrow, these different Doppler frequency components will fall into different Doppler frequency bin, which is called as Doppler spread in this paper. Obviously, the presence of Doppler spread will degrade the performance of target detection. In order to avoid Doppler spread, the maximum frequency deviation relative to carrier radar frequency \( f_0 \) must satisfy the following limitation

\[
\max_{n=0, \ldots, N-1} |c_n \Delta f| < \frac{c}{4MT_sv}
\]

#### 4.3 Effects on range resolution

From the discussion in Section 3, we know that both frequency sweep bandwidth \( \mu T_s \) and the whole frequency deviation of the transmit antenna \( \Delta f \) will affect the range resolution of the system. In this sub-section, the ambiguity function (AF) is utilised to show how these two parameters affect the range resolution of the system. The AF represented in terms of radar measurement delay and Doppler, provides a good first-order estimate of target radial velocity.

By substituting (2) into (12), we have

\[
A(\tau, f_d) = \int_{-\infty}^{+\infty} r(t) e^{i2\pi f_d t} dt
\]

The AF given by the above equation is not only dependent on time delay \( \tau \) and Doppler frequency \( f_d \), but also on target range. The dependence of AF on target range is introduced by the gating sequence for generating FMICW waveform. The AFs of various gating schemes were first examined by Khan and Mitchell [10]. Design procedures were also developed for FMICW waveforms to realise a desired AF [10]. Although the AF is dependent on target range, the range resolution is independent on it. Thus, by neglecting the effects of gating sequence and assuming the time delay introduced by the distance between transmit antennae has been well compensated, (13) is modified as

\[
A(\tau, f_d) = \int_{-\infty}^{+\infty} \left( \sum_{k=1}^{N} e^{i2\pi f_d (t + \tau_k) - (u/2)(t + \tau_k)^2} \right) \times \left( \sum_{k=1}^{N} g(t + \tau_k) e^{i2\pi f_d (t + \tau_k) - (u/2)(t + \tau_k)^2} \right)^* dt
\]
Substituting $f_0 = 0$ into (14), we have

$$A(\tau, 0) = \int_{-\infty}^{+\infty} \left( \sum_{n=1}^{N} e^{j 2 \pi f_0 (t-\tau_n)} - (u/2)(t-\tau_n)^2 \right) \right) d\tau \times \left( \sum_{k=1}^{N} e^{j 2 \pi f_k (t+\tau_n)} - (u/2)(t+\tau_n)^2 \right) d\tau \right)

= A_1(\tau, 0) + A_2(\tau, 0) \tag{15}$$

where

$$A_1(\tau, 0) = e^{j 2 \pi (\mu/2) - \mu \tau_0} \sum_{n=1}^{N} e^{-j 2 \pi f_0} \int_{-\infty}^{+\infty} e^{j 2 \pi f_0 \tau} d\tau$$

$$A_2(\tau, 0) = \sum_{n,k=1}^{N} \int_{-\infty}^{+\infty} e^{j 2 \pi f_k (t-\tau_n)} - (u/2)(t-\tau_n)^2 \right) \right) d\tau \times \left( e^{j 2 \pi f_j (t+\tau_n)} - (u/2)(t+\tau_n)^2 \right) d\tau \right)

The first term $A_1(\tau, 0)$ in (15) dominates the mainlobe characteristics of the AF, and the second term $A_2(\tau, 0)$ in (15) mainly contributes to the sidelobe characteristics of the AF. Thus, the range resolution is mainly determined by $A_1(\tau, 0)$. After some derivations and manipulations, $A_1(\tau, 0)$ can be written as

$$A_1(\tau, 0) = e^{j 2 \pi (\mu/2) - \mu \tau_0} f_0 \tau_0 - (N-1)(1/2)(\tau_n)^2 \right) \right) (T_n - |\tau|) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right)$$

where $f_0 = f_0 + (n-1)\Delta f_n, n = 1, \ldots, N$. has been used.

Let $A_1(\tau) = \left( \sin(\pi \mu (T_n - |\tau|)/\pi N) \right) / (\sin(\pi \mu (T_n - |\tau|)/\pi N))$ and $A_2(\tau) = \left( \sin(\pi \mu (T_n - |\tau|)/\pi N) \right) / (\sin(\pi \mu (T_n - |\tau|)/\pi N))$. As (16) suggests, the range resolution is determined by both $A_1(\tau)$ and $A_2(\tau)$. The range resolution determined by $A_1(\tau)$ and $A_2(\tau)$ is $\Delta R_1 = c/2B_1 = c/2B_1$ and $\Delta R_2 = c^2/2B_2 = c^2/2B_2$, respectively. $B_1 = \mu B_1$ denotes the frequency sweep bandwidth of the transmit antenna, and $B_2 = \mu^2 B_1$ denotes the frequency deviation of the transmit antenna relative to $f_0$. The effects of $B_1$ and $B_2$ on range resolution of the system can be divided into three cases.

Case 1: $B_1 \gg B_2$, the range resolution is more dependent on $B_1$ than on $B_2$. In other words, targets can be resolved by range transform processing (with range resolution $\Delta R_1$) but cannot be resolved by $\Delta R_2$, then the range resolution of the system is approximated to $\Delta R_1$. This conclusion is reasonable. As illustrated in Fig. 2, the synthesis processing is performed on the received signals of the same range bin with a width of $\Delta R_1$. Thus in this case $\Delta R_1$ limits the range resolution.

Case 2: $B_1 \ll B_2$, the range resolution is more dependent on $B_2$ than on $B_1$.

Case 3: $B_1 \approx B_2$, they have approximately the same effects on the range resolution.

In order to effectively utilise the signal bandwidth and alleviate the requirement of larger receiver bandwidth, a reasonable choice is $B_1 \approx B_2$.

### 4.4 Target range and azimuth angle uncoupled estimation

As described earlier, the equivalent steering vector of the received signal is

$$\mathbf{a}(R_0, \theta) = [e^{-j 2 \pi f_0 (\tau_0 - \Delta \tau_1)}, e^{-j 2 \pi f_0 (\tau_0 - \Delta \tau_2)}, \ldots, e^{-j 2 \pi f_0 (\tau_0 - \Delta \tau_n)}]^T \tag{17}$$

as (17) suggests, $\mathbf{a}(R_0, \theta)$ depends on both target range and azimuth angle. Thus target range and azimuth angle estimation by synthesis processing, which is illustrated in Fig. 2, may be coupling. The Cramer–Rao inequality provides a relatively simple lower bound on the variance of unbiased estimates [13–15]. Coupled estimates cause the Cramer–Rao lower bound (CRLB) for each estimate to be degraded. In this sub-section, we will use CRLB to find conditions for uncoupled target range and azimuth angle estimates. For multiple parameter estimation, the CRLB’s are diagonal elements of the inverse Fisher information matrix $\mathbf{F}$. In our case, the $(i, j)$th element of the $2 \times 2$ matrix $\mathbf{F}$ is defined by

$$[\mathbf{F}(\xi)]_{i,j} = E \left[ \frac{\partial \ln p(\mathbf{x}; \xi)}{\partial \xi_i} \frac{\partial \ln p(\mathbf{x}; \xi)}{\partial \xi_j} \right] \tag{18}$$

where $\mathbf{x}$ is the data vector, $\xi = [R_0, \theta]^T$ is the parameter vector and $p(\mathbf{x}; \xi)$ denotes conditional probability density function. In our case for joint estimation of two parameters

$$\text{var}(\hat{\xi}_i) \geq \frac{1}{\mathbf{F}_{i,i}} \left[ 1 - \frac{\mathbf{F}_{i,j}^2}{\mathbf{F}_{i,i} \mathbf{F}_{j,j}} \right]^{-1} \tag{19}$$

The first factor of (19) is the lower bound for the estimate of $\hat{\xi}_i$ when only this parameter is unknown. The second term is greater or equal to unity. When the second term is greater than unity the uncertainty of one parameter increases the minimum variance for the other parameter and the estimates are coupled. If $F_{i,j}, i \neq j$, equal zero, the second term is equal to unity, the estimates are said to be uncoupled. From (8) and (18), the individual expressions for $F_{i,j}$ are

$$F_{1,1} = 8 \text{SNR} \frac{(2\pi c) \sum_{n=1}^{N} (\Delta f_n)^2}{c^2} \tag{20}$$

$$F_{2,2} = 2 \text{SNR} \frac{(2\pi c) \sum_{n=1}^{N} (f_n |r_n| \sin(\theta - \phi_n))^2}{c^2} \tag{21}$$

$$F_{1,2} = 2 \text{SNR} \frac{(2\pi c) \sum_{n=1}^{N} (f_n |r_n| \sin(\theta - \phi_n))^2}{c^2} \tag{22}$$

where $\text{SNR} = |A|^2/\sigma^2$ denotes the SNR of the received signal after range transform. A necessary and sufficient condition for uncoupled estimation, namely $F_{1,2}$, equals zero is that

$$\sum_{n=1}^{N} f_n |r_n| \sin(\theta - \phi_n) = 0 \tag{23}$$

It follows that

$$\sum_{n=1}^{N} f_n |r_n| \sin(\cos \phi_n \sin \theta - \sin \phi_n \cos \theta) = 0 \tag{24}$$
For (24) to hold for all $\theta$ (target azimuth angle), we have 
\[ \sum_{n=1}^{N} f_n \Delta f_n |r_n| \cos \phi_n = \sum_{n=1}^{N} f_n \Delta f_n |r_n| \sin \phi_n = 0 \]
or in complex notation
\[ \sum_{n=1}^{N} f_n \Delta f_n |r_n| e^{i\phi_n} = 0 \]  \(25\)

Let $r'_n = f_n \Delta f_n |r_n|$, then (25) can be written as 
\[ \sum_{n=1}^{N} r'_n e^{i\phi_n} = 0 \]  \(26\)

As (26) suggests, there are many transmit array geometries that satisfy (26). A straightforward choice is pseudo-circular array geometry. The name comes from that $r'_n$ is not the distance between the $n$th transmit antenna and reference point, the actual distance can be found by the relation $r'_n = f_n \Delta f_n |r_n|$. For the pseudo-circular array geometry, $N$ antennae are evenly spaced on a ring of radius $r$ centred about the origin of the Cartesian coordinate system, which is shown in Fig. 1. For this geometry, (26) can be written as 
\[ r \sum_{n=1}^{N} e^{i2\pi(n-1)/N} = 0 \]  \(27\)

where $r = r'_1 = r'_2 = \ldots = r'_N$. The sum of the above equation is a geometric series that equals zero for $N \geq 3$. Thus, uncoupled parameters estimation can be obtained.

### 4.5 Summary

As discussed earlier, the transmit frequencies affect the performance of the system in many respects. It is therefore critical that the transmit frequencies should be deliberately selected. In practice, by taking into account all the effects of the transmit frequencies on system performance, a compromise solution may be obtained to achieve an overall best system performance.

### 5 Numerical examples

In this section, we present some examples to demonstrate the signal processing scheme and the effects of various
combinations of \(B_1\) and \(B_2\) on range resolution of the system. The following system parameters are used in simulations, the transmit antenna array uses pseudo-circular array geometry with element number \(N = 12\), the position vector of each transmit antenna meets the requirement of (27), \(f_0 = 7\) MHz, \(T = 1\) s, \(M = 100\), and a single reception antenna located at the origin of the Cartesian coordinate system as shown in Fig. 1.

The first example is chosen to demonstrate the synthesis processing and the coherent integration output of a target located at distance \(R_0 = 120\) km, with azimuth angle \(\theta = 85^\circ\), and radial velocity \(v = -4\) m/s. Fig. 3 shows the output of the synthesis processing. As shown in Figs 3a and b, a large peak exists at the position corresponding to target range and azimuth angle, at 3-D range–azimuth angle profile. Figs. 3c and d show a slice of the 3-D range–azimuth angle profile with \(\theta = 85^\circ\) and \(R_0 = 120\) km, respectively. As illustrated in Fig. 3 target range and azimuth angle can be estimated from the output of synthesis processing. Fig. 4 shows the outputs of the coherent integration, which is performed on the output of the synthesis processing at range \(R_0 = 120\) km. As illustrated in Figs. 4a and b, a large peak also exists at the position corresponding to target Doppler and azimuth angle, at 3-D Doppler–azimuth angle profile. Fig. 4c shows a slice of 3-D Doppler–azimuth angle profile with \(\theta = 85^\circ\). As demonstrated in Fig. 4, target Doppler frequency can be estimated from the output of coherent integration.

The second example is to compare the range resolution obtained by range transform only and that achieved after synthesis processing, with various combinations of \(B_1\) and \(B_2\). Fig. 5a shows the range transformed (dashed line) and the synthesis processing (solid line) results of a stationary target located at distance \(R_0 = 45\) km with \(B_1 = 30\) kHz and \(B_2 = 15\) kHz. As demonstrated in Fig. 5a, the range resolution of the system is \(\Delta R \approx 5\) km which approximates to that of \(\Delta R_1 = 5\) km. Fig. 5b shows the results of the same target with \(B_1 = 30\) kHz and \(B_2 = 60\) kHz. As illustrated in Fig. 5b, the range resolution of the system is \(\Delta R \approx 2.5\) km which approximates to that of \(\Delta R_2 = 2.5\) km. Fig. 5c shows the results of the same target with \(B_1 = 30\) kHz and \(B_2 = 30\) kHz. As illustrated in Fig. 5c, the range resolution of the system is \(\Delta R \approx 3.6\) km, which is determined by the product of two sinc functions each having a range resolution of about 5 km. Namely, both \(B_1\) and \(B_2\) have approximately the same effects on the range resolution. The results shown in Fig. 5 well confirm the analysis in Section 4.3.
6 Conclusions and future work

A novel HFSWR based on SIAP is proposed in this paper. The equivalent transmit beam patterns are formed by proper processing the received signals. It has advantages over the conventional HFSWR for both military and civil applications, such as good ECCM performance, high agility and manoeuvrability, good survivability, multiple fix or moving receive platforms sharing one transmit antenna array and so on. The signal processing scheme to obtain equivalent transmit beam patterns is investigated in detail. Considerations for system parameter selection to achieve overall best system performance are proposed and discussed. In particular, a necessary and sufficient condition on choosing the transmit array geometry and transmit frequencies, which ensures that target range and angle estimates are uncoupled, is presented.

This paper mainly concerns the signal processing issues of the proposed HFSWR. For practical implementation, several problems should be further studied, such as (i) synchronisation between transmit antenna array and receive platforms, (ii) spatial–temporal super resolution algorithms to improve target detection performance under multiple targets environments, (iii) clutter cancellation schemes to alleviate sea clutter spectrum spread effects introduced by receive platforms motion, (iv) data fusion techniques to further improve the detection performance and measure accuracy by combining the data from multiple receive platforms.

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8 References


Fig. 5 Comparison of range resolutions obtained by range transform and synthesis processing with various $B_1$ and $B_2$ combinations

$a B_1 = 30$ kHz and $B_2 = 15$ kHz

$b B_1 = 30$ kHz and $B_2 = 60$ kHz

$c B_1 = 30$ kHz and $B_2 = 30$ kHz
8 Zhang, Q.W.: ‘Study of the performances of synthetic impulse and aperture radar system’, PhD Dissertation, Xidian University, Xi’an, People’s Republic of China, 1994