Contourlet-based image denoising algorithm using directional windows

Z.-F. Zhou and P.-L. Shui

Contourlet is a new effective signal representation tool in many image applications. Proposed is a contourlet-based image denoising algorithm using directional windows which takes advantage of the captured directional information of the natural images. Experiments show that the proposed algorithm achieves better performance than other contourlet-based image denoising algorithms.

Introduction: The separable two-dimensional wavelet has been widely used in many image processing applications. But it has limited ability in capturing the directional information of the natural images. To overcome this deficiency, Do and Vetterli [1, 2] developed the contourlet transform based on a multiscale and multidirectional filter bank which can capture nearly arbitrarily directional information of the natural images. It is implemented by Laplacian pyramid and two-dimensional directional filter banks that can simultaneously hold multiresolution, localisation, nearly critical sampling, flexible directionality and anisotropy. Based on a detailed study of the statistical features of the contourlet coefficients, Po and Do [3] proposed a contourlet hidden Markov tree (HMT) model for image denoising and texture retrieval applications. It makes good use of the inter-scale and intra-scale dependency of the contourlet coefficients and lacks in utilising the captured directional information of the natural images. In this Letter we propose a simple but effective contourlet-based image denoising algorithm which utilises the captured directional information of the natural images by directional windows. Experiments show that our algorithm can obtain higher peak signal-to-noise ratio.

Fig. 1 ‘Octagon’ image; example of frequency partition; four directional subbands of contourlet coefficients; corresponding four directional windows

a ‘Octagon’ image
b Example of frequency partition
c Four directional subbands of contourlet coefficients
d Corresponding four directional windows

Directional windows: Similar to wavelet, contourlet can decompose the image into different scales. But unlike the wavelet which can only decompose each scale into two directions, contourlet can decompose each scale into any arbitrarily power of two’s number of directions and different scales can be decomposed into different numbers of directions. Figs. 1a–e show the ‘octagon’ image, an example frequency partition of the contourlet transform and only one-scale four directional subbands of the contourlet decomposition, respectively. It can be seen that the energy clusters in each directional subband are be seen that the energy clusters in each directional subband are

\[
W(r, a, \theta) = \left\{ (m, n) : \left( \frac{\sin^2 \theta}{a} + a^2 \cos^2 \theta \right)m^2 + \frac{a^4 - 1}{a^2} \sin 2\theta mn + \left( \frac{\cos^2 \theta}{a} + a^2 \sin^2 \theta \right)n^2 \leq r^2 \right\}
\]

where \( r, a \geq 0 \) and \( \theta \in [-\pi, \pi] \), \( r, a \) and \( \theta \) determine the window’s size, shape and the principal axis direction, respectively. Fig. 1d gives the four directional windows that correspond to each directional subband of the contourlet decomposition in Fig. 1c. Particularly, if we decompose one scale into only two directions (wavelet decomposition), the windows used in the horizontal and vertical subband are \( W(r, a, 0) \) and \( W(r, a, \pi/2) \). But the energy distribution in the diagonal subband is along the diagonal and antidirectional directions, so we use the cross-shaped window which is defined as:

\[
W(r, a) = \{ (m, n) : \min[a^2p^2 + q^2, a^2q^2 + p^2] \leq a^2r^2 \}
\]

where \((p, q) = (m+n, m-n)\) and \(a \geq 1\).

Proposed denoising algorithms: The local Wiener filtering in the wavelet domain is an effective image denoising method [4–6]. Because the contourlet has the similar characteristic as the wavelet, so we can straightforwardly extend the local Wiener filtering into the contourlet domain. Assume that an image is corrupted by additive stationary Gaussian noise of zero-mean and variance \(\sigma^2\). The observed image is represented in the contourlet domain by

\[
y_k(i, j) = \hat{s}_k(i, j) + e_k(i, j).
\]

where \(y_k(i, j), \hat{s}_k(i, j)\) and \(e_k(i, j)\) are the contourlet coefficients in the \(k\)th directional subband of the observed noisy image, noise-free image and noise, respectively. For simplicity, the additional indexes for level/scale are ignored.

The local Wiener filtering in the contourlet domain includes two main steps. First, the signal variance of each noisy contourlet coefficient is estimated by the local average

\[
\hat{\sigma}^2_k(i, j) = \max \left\{ 0, \frac{1}{\#W_k} \sum_{(p, q) \in W_k} y^2_k(i+p, j+q) - \sigma^2 \right\}
\]

where \(W_k\) and \(\#W_k\) denote the directional window and its size, respectively. For a given scale and \(2^d\) directional decomposition, from Fig. 1, it can be seen that the window used in the \(k\)th oriented subband is

\[
W_k = \left\{ (r, a, \theta) : \theta = \frac{(k-1)}{2^{d-1}} \pi \right\}
\]

where the initial angle \(\theta_0 = ((3/4) - (1/2))\pi\). Secondly, the contourlet coefficients of the signal are estimated by the local Wiener filtering

\[
\hat{s}_k(i, j) = \frac{\hat{\sigma}^2_k(i, j)}{\sigma^2_k(i, j)} y_k(i, j)
\]

where the initial angle \(\theta_0 = ((3/4) - (1/2))\pi\). Secondly, the contourlet coefficients of the signal are estimated by the local Wiener filtering

\[
\hat{s}_k(i, j) = \frac{\hat{\sigma}^2_k(i, j)}{\sigma^2_k(i, j)} y_k(i, j)
\]

Experiment results: We tested our algorithm on ‘Lena’ and ‘Barbara’ images to make a comparison with other contourlet-based image denoising algorithms. For the contourlet transform, we use the 9-7 biorthogonal filter for the multiscale decomposition stage and CD filters [3] for the multidirectional decomposition stage. The finest and second finest scales are partitioned into eight directional subbands, and the two next coarser scales are partitioned into four directional subbands. The noise variance is estimated using a Monte Carlo method [3]. We use the elliptic directional window with \(r = (6,6,4,4)\) and \(a = 5.1\) from the finest scale to the coarsest scale, respectively. The output PSNRs for the ‘Lena’ and ‘Barbara’ images are listed in the last row of Table 1 with the best results highlighted in bold. From Table 1, it can be seen that our algorithm achieves the...
maximal PSNR output among other contourlet-based image denoising algorithms.

**Table 1:** Comparison of several denoising algorithms.

<table>
<thead>
<tr>
<th>Image</th>
<th>Lena</th>
<th>Barbara</th>
<th>Lena</th>
<th>Barbara</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_1$</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>30</td>
</tr>
<tr>
<td>Wavelet HMT</td>
<td>28.35</td>
<td>27.21</td>
<td>25.89</td>
<td>25.11</td>
</tr>
<tr>
<td>Contourlet HMT [3]</td>
<td>28.18</td>
<td>27.00</td>
<td>26.04</td>
<td>25.27</td>
</tr>
<tr>
<td>Proposed algorithm</td>
<td>28.77</td>
<td>27.47</td>
<td>26.46</td>
<td>26.34</td>
</tr>
</tbody>
</table>

**Conclusions:** A simple and effective contourlet-based image denoising algorithm using directional windows is proposed. For the next step we will use the directional windows in the nonsubsampled contourlet [7] case.

**Acknowledgment:** This work is supported by the NSF (project no. 60472086) of the People’s Republic of China.

© The Institution of Engineering and Technology 2007
12 October 2006
Electronics Letters online no: 20073166
doi: 10.1049/el:20073166

Z.-F. Zhou and P.-L. Shui (National Laboratory of Radar Signal Processing, Xidian University, Xi’an 710071, People’s Republic of China)
E-mail: ever817@126.com

**References**