Robust adaptive beamforming in nested array

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A R T I C L E   I N F O

Article history:
Received 2 December 2014
Received in revised form
9 February 2015
Accepted 28 February 2015
Available online 10 March 2015

Keywords:
Robust adaptive beamforming
Nested array
Interference-plus-noise covariance matrix reconstruction
Steering vector estimation

A B S T R A C T

In this communication, we consider the problem of designing robust adaptive beamformer in the newly proposed nested array, where we maintain distortionless response towards the true desired signal and suppress more interferences than the number of actual physical sensors. The essence of our proposed method is to reconstruct the interference-plus-noise covariance matrix and estimate the true desired signal steering vector. The reconstruction process is performed by projecting the spatially smoothed matrix into interference subspace. The estimation process is performed by solving an optimization problem based on minimizing the beamformer sensitivity and enforcing the estimate not to converge to any of the interference steering vectors. We also show that the constructed optimization problem can be efficiently computed at a comparable cost with that of the standard Capon beamformer (SCB). The effectiveness of the proposed method is verified through numerical simulations.

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1. Introduction

Nested array is a novel antenna array geometry proposed recently, which is obtained by combining two or more uniform linear arrays (ULAs) with increasing spacing. The superiority of nested array compared with the traditional ULA or sparse array is that we can achieve $O(N^2)$ degrees of freedom (DOF) using only $O(N)$ sensors, which makes it possible for nested array to resolve more sources or suppress more interferences than the number of actual physical sensors. To achieve the extended DOF provided by nested array, the vectorized received signal covariance matrix is utilized. In this communication, we concentrate on the aspect of robust adaptive beamformer design in nested array only.

The existing adaptive beamforming method for nested array is the minimum variance distortionless response (MVDR) beamformer proposed by Pal et al. [1]. It should be pointed out that the spatially smoothed matrix corresponding to a longer difference co-array, instead of the sample covariance matrix as is the convention, is utilized in this beamformer to gain the full DOF offered by nested array. However, the MVDR beamformer is known to suffer from performance degradation when the desired signal contaminates the received training data or mismatch between the presumed and the actual desired signal steering vector exists, especially at the high input signal-to-noise ratio (SNR) level [2]. To alleviate this problem, robust approach to adaptive beamforming in nested array is required. Considering state-of-the-art techniques [3–14], we know that the actual desired signal steering vector can be estimated by utilizing various methods based on convex optimization, such as the worst-case method [6], the sequential quadratic programming (SQP) method [12] and the method proposed in [14]. Noting that all of the convex optimization problems to be solved in these methods share a similar structure, i.e., maximizing the Capon beamformer output power and considering the uncertainty set for the desired signal steering vector. Nevertheless, these beamformers solely utilize the contaminated received signal covariance matrix instead of the required interference-plus-noise covariance matrix.

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http://dx.doi.org/10.1016/j.sigpro.2015.02.027
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covariance matrix, and according to Rübsamen and Pesavento [15], maximizing the beamformer output power may lead to a poor suppression of interferences and noise. Hence, as a result, the robustness against desired steering vector mismatch may not be sufficient.

In this communication, we propose a robust beamforming method in nested array based on interference-plus-noise covariance matrix reconstruction and steering vector estimation. The interference-plus-noise covariance matrix is reconstructed by projecting the spatially smoothed matrix into the interference subspace, thus the unwanted desired signal component is eliminated. The true desired signal steering vector is estimated by solving a novel convex optimization problem, which is based on minimizing the beamformer sensitivity to model errors and constraining the estimate not to converge to any interference steering vectors or their linear combinations. We also prove that the proposed optimization problem can be efficiently solved using Lagrange multiplier methodology, which has a comparable computational complexity with that of the SCB. Numerical simulation results are provided to validate the effectiveness of our method.

Notations: we use bold upper-case and lower-case letters to represent matrices and vectors, respectively. The superscripts \((\cdot)^T\), \((\cdot)^H\) and \((\cdot)^{-1}\) denote the transpose, Hermitian transpose and inverse operators, respectively. \(\text{diag}(\cdot)\) is the diagonalization operator. \(\mathbb{R}\) is used to denote the set of real numbers. \(j\) is reserved for the imaginary unit \(\sqrt{-1}\).

2. Signal model

Without loss of generality, we consider an \(M\)-element linear nested array, which is a concatenation of two ULAs: the inner ULA has \(M_1\) sensors with intersensor spacing \(d_i\) and the outer ULA has \(M_2\) sensors with intersensor spacing \(d_0 = (M_1 + 1)d_i\), as shown in Fig. 1. We assume \(K\) independent narrowband sources impinging on this array from farfield region at directions \(\{\theta_k, k = 1, 2, ..., K\}\), then the received signal can be expressed as

\[
y(t) = As(t) + n(t)
\]

where \(y(t) = [y_1(t), y_2(t), ..., y_M(t)]^T\) is the received signal vector at time \(t\), \(s(t) = [s_1(t), s_2(t), ..., s_K(t)]^T\) is the source signal waveform vector and \(s_k(t) \sim \mathcal{N}(0, \sigma^2_k)\), \(n(t) = [n_1(t), n_2(t), ..., n_M(t)]^T\) is the white Gaussian noise vector with power \(\sigma^2_n\) and uncorrelated with the sources. Let \(a(\theta_k) = \{e^{2\pi i n r_j \sin \theta_k/d_0}, n = 1, 2, ..., M\}\) denote the steering vector of the \(k\)th signal, where \(\lambda_0\) is the carrier wavelength and \(\{r_j\}_{n = 1, 2, ..., M} = \{0, 1, ..., M_1 - 1, M_1, 2(M_1 + 1) - 1, ..., M_2(M_1 + 1)\} + d_0\) is a scalar vector that contains the position information of all sensors. Then the manifold matrix \(A\) can be written as

\[
A = [a(\theta_1), a(\theta_2), ..., a(\theta_K)]
\]

Collecting \(Q\) snapshots and averaging them through time, then the sample covariance matrix can be expressed as

\[
\hat{R} = \frac{1}{Q} \sum_{t=1}^{Q} y(t)y^H(t) \approx ARAR^H + \sigma^2_nI
\]

with \(R = \text{diag}\{\sigma^2_1, \sigma^2_2, ..., \sigma^2_K\}\). Vectorizing \(\hat{R}\) in (3), we can get a long vector. It can be noted that some elements appear more than once in this vector. By removing these repeated rows and sorting them so that the \(i\)th row corresponds to the sensor located at \((-M + 1)d_i\) with \(M = M^2/4 + M/2\), then a new vector \(z\) is obtained

\[
z = BP + \sigma^2_n e
\]

where

\[
B = [b(\theta_1), b(\theta_2), ..., b(\theta_K)]
\]

with

\[
b(\theta_k) = \left[ e^{-j2\pi d_i (\sin \theta_k/d_0)} e^{-j2\pi d_i (\sin \theta_k/d_0)} ..., e^{-j2\pi d_i (\sin \theta_k/d_0)} \right]^T,
\]

\[
P = [\sigma^2_1, \sigma^2_2, ..., \sigma^2_K]^T, \quad e \in \mathbb{R}^{(2M-1) \times 1}
\]

is a vector of all zeros except for a 1 at the center position. Comparing (4) with (1), we can observe that \(z\) in (4) behaves like the signal received by a longer difference co-array of the original array, whose sensor positions can be precisely determined by the distinct values in the set \(\{r_i - r_j\}_{1 \leq i, j \leq M}\). In order to exploit the increased DOF provided by nested array, we can apply the spatial smoothing method, which has been proposed in [1], to \(z\) for getting a full rank covariance matrix \(\hat{R}\) corresponds to the co-array

\[
\hat{R} = \frac{1}{M^2/4 + M/2} \sum_{i=1}^{M^2/4 + M/2} z_i z_i^H = \frac{1}{M^2/4 + M/2} (B_i R B_i^H + \sigma^2_n I)^2
\]

where \(z_i\) corresponds to the \((M^2/4 + M/2 - i + 1)\)th to \(((M^2/2)/2 + M - i + 1)\)th rows of \(z\), and \(B_i\) is a manifold matrix consists of the last \(M\) rows of \(B\). After obtaining \(\hat{R}\), the MVDR beamformer weight vector of nested array can be calculated as [1]

\[
w_{\text{MVDR}} = R^{-1} f_{\hat{b}} b_1
\]

where \(R\) is called spatially smoothed matrix and \(R = \sqrt{M^2/4 + M/2} \hat{R}^{1/2}, \quad \hat{b}_1 = [1, e^{2\pi i n r_1 \sin \theta_1/d_0}, ..., e^{2\pi i (n-1)r_1 \sin \theta_1/d_0}, d_0]

![Fig. 1. The geometry configuration of a nested array with \(M\) sensors.](image-url)
sin $\hat{\theta}_1/\lambda_0^T$ denotes the presumed desired signal steering vector with $\hat{\theta}_1$ being the presumed direction-of-arrival (DOA). Noticing that $w_{\text{MVDR}}$ can be viewed as working with the difference co-array, whose sensor positions set are $\{0, 1, \ldots, M-1\}$, rather than the original array. Consequently, we can suppress more interferences than the number of actual physical sensors since $M > M$.

3. Proposed robust beamforming method

It can be seen that $w_{\text{MVDR}}$ in (6) is not a robust beamformer weight vector since $\hat{R}$ is always contaminated by the desired signal component and $\hat{b}_1$ may deviate from the true desired signal steering vector. To combat these drawbacks, in this section, we propose a robust beamformer with interference-plus-noise covariance matrix reconstruction and desired signal steering vector estimation.

Based on the relationship between $R$ and $\hat{R}$, we can rewrite $R$ in (6) as

$$R = \sum_{k=1}^{K} \sigma_k^2 b_k(\phi_k)b_k^H(\phi_k) + \sigma_n^2 I$$

where $b_k(\phi_k) = [1, e^{j2\pi d_1 \sin \phi_k/\lambda_0}, \ldots, e^{j2\pi (M-1)d_1 \sin \phi_k/\lambda_0}]^T$. Vectorizing $R$ in (7), we get the following long vector:

$$\text{vec}(R) = \sum_{k=1}^{K} \sigma_k^2 \text{vec}(b_k(\phi_k)b_k^H(\phi_k)) + \sigma_n^2 \mathbf{1}^T$$

where vec($) is a vector obtained by stacking the columns of the argument on top of each other and $\mathbf{1}^T = [e_1^T, e_2^T, \ldots, e_M^T]^T$ with $e_i$ being a vector of all zeros except a 1 at the $i$th position. Assuming a priori knowledge about the angular sector in which the desired signal is located (i.e., $\theta$) is known and defining 'correlation vector' which belongs to direction $\theta$ as $d(\theta) = \text{vec}(b(\theta)b^H(\theta))$. Then according to (8), we know that the desired vectorized interference-plus-noise covariance matrix lies in the subspace of $F$, which can be calculated as $F = \int_{\Delta} d(\phi)d(\phi)^H d\phi$ with $\hat{\theta}$ is the complement of the sector $\theta$. Let $F = U\Lambda U^H$ denote the eigenvalue decomposition of $F$, where $U$ and $\Lambda$ are unitary and diagonal matrices constituted by eigenvectors and eigenvalues of $F$, respectively. Then the dominant subspace of $F$ can be formed by

$$U_{IS} = [U_1 U_2 \ldots U_l]$$

where $[U_l]_{l=1}^L$ are the principal eigenvectors of $F$.

In order to eliminate the unwanted part of desired signal, vec($\hat{R}$) in (8) should be projected into a subspace that collects information about the interference signal. The projection matrix $P$ is given by

$$P = U_{IS} (U_{IS}^H U_{IS})^{-1} U_{IS}^H$$

Pre-multiplying vec($\hat{R}$) with $P$, we can reconstruct the vectorized interference-plus-noise covariance matrix vec($\hat{R}_{i+n}$) as

$$\text{vec}(\hat{R}_{i+n}) = P \text{vec}(\hat{R}) + \sigma_n^2 (I - P) \mathbf{1}^T$$

Once vec($\hat{R}_{i+n}$) is obtained, $\hat{R}_{i+n}$ can be easily recovered by permuting vec($\hat{R}_{i+n}$) to an $M \times M$ matrix as the column’s order. We also note that $\sigma_n^2$ in (11) can be determined as the smallest eigenvalue of $\hat{R}$ in (7).

For estimating the true desired signal steering vector, the worst-case method, the SQP method or the method proposed in [14] can be applied. From the mathematical analysis and simulation results presented in [14], we can observe that among all these methods, the proposed in [14] utilizes the least prior information and gets the smallest estimation error. The basic idea of this method is to maximize the beamformer output power, and enforce the estimate not to converge to any of the interference steering vectors or their linear combinations by means of the following constraint

$$b_1^H \hat{C} b_1 \leq \Delta_0$$

(12)

where $\hat{b}_1$ is the estimated steering vector, $\hat{C}$ is calculated by $\int_{\Delta} \hat{b}_1(\theta) \hat{b}_1^H(\theta) d\theta$ with $\hat{b}_1(\theta)$ has the same structure as $b_1$ in (6), except for replacing $\hat{\theta}_1$ with $\theta$, $\Delta_0 = \max_{\theta \in \Delta} b_1^H(\theta) \hat{C} b_1(\theta)$.

However, as stated in the previous section, all of the techniques mentioned above do not provide sufficient robustness against signal model mismatches. In addition, following the analysis in [15], we know that the robustness of these existing beamformers can be enhanced by replacing their objective function with the minimizing beamformer sensitivity $T_{sr}$, which is defined as $T_{sr} = \|w_1f_1^\dagger b_1^\dagger b_1^H\|^2$ with $w$ is the beamformer weight vector. Thus, by using the reconstructed $\hat{R}_{i+n}$ and combing the constraint (12) with the requirement for minimizing $T_{sr}$ together, we propose a novel robust beamforming method formulated as

$$\min_{w, b_1} w^H w / |w_1^H b_1^H|^2$$

s.t. $w = \hat{R}_{i+n}^{-1} b_1^H \hat{b}_1^H \hat{C} b_1 \leq \Delta_0$ (13)

Substituting the equality constraint of (13) back into the objective function yields

$$\min_{b_1} b_1^H \hat{R}_{i+n}^{-1} b_1 / |b_1^H \hat{R}_{i+n}^{-1} b_1|^2$$

s.t. $b_1^H \hat{C} b_1 \leq \Delta_0$ (14)

It can be seen from (14) that the objective function is invariant w.r.t. the scaling of $\hat{b}_1$. Thus, for simplicity, we can scale $\hat{b}_1$, such that $b_1^H \hat{R}_{i+n}^{-1} b_1 = 1$, and the original problem (14) can be transformed to

$$\min_{b_1} b_1^H \hat{R}_{i+n}^{-2} b_1$$

s.t. $b_1^H \hat{C} b_1 \leq \Delta_0$ (15)

The optimization problem (15) can be readily solved using the approach presented in the following proposition, which has a comparable computational cost (viz., $O(M^3)$) with that of the SCB.
Proposition: The optimal minimizer of (15) can be calculated as
\[ \hat{b}_1 = \frac{(\hat{R}_{i+n}^{-1} + \lambda \hat{R}_{i+n} \hat{C})^{-1} \hat{b}_1}{\hat{b}_1^H (1 + \lambda \hat{R}_{i+n} \hat{C} \hat{R}_{i+n})^{-1} \hat{b}_1} \] (16)
where the value of \( \lambda \) can be determined as the solution to the following constraint equation:
\[ \frac{\hat{b}_1^H (1 + \lambda \hat{R}_{i+n} \hat{C} \hat{R}_{i+n})^{-1} \hat{b}_1}{\hat{b}_1^H (1 + \lambda \hat{R}_{i+n} \hat{C} \hat{R}_{i+n})^{-1} \hat{b}_1} = \Delta_0 \] (17)

Proof: The optimization problem (15) can be efficiently solved using the Lagrange multiplier method, which is based on the Lagrangian function
\[ L(\hat{b}_1, \lambda, \mu) = \hat{b}_1^H \hat{R}_{i+n}^{-1} \hat{b}_1 + \lambda (\hat{b}_1^H \hat{C} \hat{b}_1 - \Delta_0) + \mu (\hat{b}_1 - \hat{b}_1^H \hat{R}_{i+n}^{-1} \hat{b}_1 + 2) \] (18)
where \( \lambda \) and \( \mu \) are the real-valued Lagrange multipliers with \( \lambda \geq 0 \) and \( \mu \) being arbitrary. We note that (18) can be rewritten as
\[ L(\hat{b}_1, \lambda, \mu) = \frac{[\hat{b}_1 - \mu (\hat{R}_{i+n}^{-1} + \lambda \hat{R}_{i+n} \hat{C})^{-1} \hat{b}_1]^H [\hat{b}_1 - \mu (\hat{R}_{i+n}^{-1} + \lambda \hat{R}_{i+n} \hat{C})^{-1} \hat{b}_1]}{[\hat{b}_1 - \mu (\hat{R}_{i+n}^{-1} + \lambda \hat{R}_{i+n} \hat{C})^{-1} \hat{b}_1]^H \hat{b}_1 - \lambda \Delta_0 + 2 \mu} \] (19)

Hence, the unconstrained minimization of \( L(\hat{b}_1, \lambda, \mu) \) w.r.t. \( \hat{b}_1 \), for fixed \( \lambda \) and \( \mu \), is given by
\[ \hat{b}_1(\lambda, \mu) = \mu (\hat{R}_{i+n}^{-1} + \lambda \hat{R}_{i+n} \hat{C})^{-1} \hat{b}_1 \] (20)

It can be easily derived from (19) and (20) that
\[ L_1(\lambda, \mu) = L(\hat{b}_1(\lambda, \mu), \lambda, \mu) = -\mu^2 \hat{b}_1^H (1 + \lambda \hat{R}_{i+n} \hat{C} \hat{R}_{i+n})^{-1} \hat{b}_1 \]
\[ -\lambda \Delta_0 + 2 \mu \leq \hat{b}_1^H \hat{R}_{i+n}^{-2} \hat{b}_1 \leq \Delta_0 \] for any \( \hat{b}_1 \) satisfying \( \hat{b}_1^H \hat{C} \hat{b}_1 \leq \Delta_0 \) (21)

Setting the derivative of \( L_1(\lambda, \mu) \) w.r.t. \( \mu \) to zero yields
\[ \hat{\mu} = \frac{1}{\hat{b}_1^H (1 + \lambda \hat{R}_{i+n} \hat{C} \hat{R}_{i+n})^{-1} \hat{b}_1} \] (22)
and
\[ L_2(\lambda) \triangleq L_1(\lambda, \hat{\mu}) = \frac{1}{\hat{b}_1^H (1 + \lambda \hat{R}_{i+n} \hat{C} \hat{R}_{i+n})^{-1} \hat{b}_1} - \lambda \Delta_0 \] (23)

Setting the derivative of \( L_2(\lambda) \) w.r.t. \( \lambda \) to zero gives the expression for (17). Substituting (22) in (20) yields the expression for (16). Consequently, once \( \hat{\lambda} \) has been computed, (16) allows to efficiently compute \( \hat{b}_1 \) using only \( O(\hat{M}^3) \) flops. This completes the proof.

Remark: To simplify the operation involving matrix inversion, we rewrite (17) using the eigendecomposition of \( \hat{R}_{i+n} \hat{C} \hat{R}_{i+n} \), that is, \( \hat{R}_{i+n} \hat{C} \hat{R}_{i+n} = \hat{V} \hat{R} \hat{V}^H \), where the unitary matrix \( \hat{V} = [V_1, V_2, \ldots, V_M] \) contains the orthonormal eigenvectors, and the diagonal matrix \( \hat{\Gamma} = \text{diag}(\gamma_1, \gamma_2, \ldots, \gamma_M) \) contains the eigenvalues with \( \gamma_1 \geq \gamma_2 \geq \ldots \geq \gamma_M \). Let \( \hat{z} = \hat{V}^H \hat{b}_1 \) and \( \hat{z}_m \) denote the \( m \)th element of \( \hat{z} \), then (17) can be transformed to
\[ \sum_{m=1}^{M} \frac{|\hat{z}_m|^2}{\gamma_m + \Gamma_0} = \Delta_0 \]
(24)

It can be easily shown that the left side of (24) is a monotonically decreasing function of \( \hat{\lambda} \). Let \( f(\hat{\lambda}) \) represent the left side of (24), it is clear that
\[ \lim_{\hat{\lambda} \to 0} f(\hat{\lambda}) = \left(1/\hat{M}^2\right) \sum_{m=1}^{M} \gamma_m |\hat{z}_m|^2 \approx \left(\alpha_1/\hat{M}^2\right) \sum_{m=1}^{M} |\hat{z}_m|^2 = \alpha_1/\hat{M} \]

Table 1
The proposed robust adaptive beamforming algorithm.

**Input:** physical sensor number \( M \), virtual sensor number \( \hat{M} \), observed snapshots \( \{\mathbf{y}(t)\} \) and a positive definite matrix of the form \( \mathbf{C} = \int b(t) b^H(t) \mu dt \).

**Step 1:** Compute the sample covariance matrix \( \hat{R} \) from [3].

**Step 2:** Vectorize \( \hat{R} \) and remove the repeated rows, in order to get a long vector \( \mathbf{z} \).

**Step 3:** Apply the spatial smoothing technique to \( \mathbf{z} \) for getting a full rank covariance matrix \( \hat{R} \), then the spatially smoothed matrix \( \hat{R} = \sqrt{M^2/4 + M/2\hat{k}^{1/2}} \) is obtained.

**Step 4:** Reconstruct the interference-plus-noise covariance matrix \( \hat{R}_{i+n} \) based on (11).

**Step 5:** Perform the eigendecomposition on \( \hat{R}_{i+n} \hat{C} \hat{R}_{i+n} \) as \( \hat{R}_{i+n} \hat{C} \hat{R}_{i+n} = \hat{V} \hat{R} \hat{V}^H \).

**Step 6:** Solve (24) for \( \hat{\lambda} \) by a Newton’s method.

**Step 7:** Use the \( \hat{\lambda} \) obtained in step 6 to estimate the desired signal steering vector \( \hat{b}_1 \), which is given by (25). Note that the inverse of the diagonal matrix \( \hat{I} + \hat{\lambda} \hat{R} \) is easily computed and \( \hat{V}^H \hat{b}_1 \) is available from step 5.

**Step 8:** Scale \( \hat{b}_1 \) as \( \hat{b}_1 = \sqrt{\hat{M}} \hat{b}_1 \) so that its norm is equal to \( \sqrt{\hat{M}} \).

**Output:** the robust beamformer weight vector \( \mathbf{w}_\text{rob} \), which can be calculated as \( \mathbf{w}_\text{rob} = \hat{R}_{i+n}^{-1} \hat{b}_1 / \hat{b}_1^H \hat{R}_{i+n}^{-1} \hat{b}_1 \).
and
\[
\lim_{\lambda \to \infty} \left( \lambda \right) = \frac{1}{\sum_{m=1}^{M} \frac{1}{\sigma_m^2} \sum_{m=1}^{M} \frac{|z_m|^2}{\sigma_m^2}} = \frac{1}{\sum_{m=1}^{M} |z_m|^2} = \gamma \frac{1}{\lambda}
\]

where \(\alpha\) and \(\beta\) are some scaling constants with \(1 < \alpha < M\) and \(\beta < M\). Assuming \( \hat{\mathbf{C}} \) has \( \mathcal{P} \) principal eigenvalues and \( \lambda_{j+1} \) is the \((j+1)\)th largest eigenvalue of \( \hat{\mathbf{C}} \), then according to Khabazzi-basmenj et al. \[14\], we know that \( \Delta_0 \leq M_{\gamma_{j+1}} \). Consequently, it follows that \((\alpha \gamma_{j+1}/\mathcal{P}) > \Delta_0\) and \((\gamma_{1}/\lambda) < \Delta_0\) since \(\alpha \gamma_{j+1} > \mathcal{P} \) and \(\gamma_{1} < \mathcal{P} \). Hence, there is a unique solution \( \hat{\lambda} \in (0, \infty) \) to (24), which can be obtained efficiently via a Newton’s method. Following the similar analysis to derive (24), we can rewrite (16) as
\[
\hat{b}_1 = \frac{\hat{R}_{i+n} \mathbf{V} (I + \hat{\lambda} \mathbf{I})^{-1} \mathbf{v}^H \hat{b}_1}{\hat{b}_1 \mathbf{V} (I + \hat{\lambda} \mathbf{I})^{-1} \mathbf{v}^H \hat{b}_1}
\]

Up to now, the proposed robust beamforming method can be summarized in Table 1.

From a complexity point of view, the main computational cost our method is due to the calculation of \( \hat{R}_{i+n} \) in step 1 and the eigendecomposition of \( \hat{R}_{i+n} \hat{\mathbf{C}} \hat{R}_{i+n} \) in step 2, both of which require \( O(M^3) \) flops. Additionally, compared to the previous techniques in \[6,12,14\] which have complexity equal or higher than \( O(M^{3.5}) \), our algorithm has a lower cost \( O(M^{3}) \).

4. Simulation results

In our simulations, we consider a two level nested array contains \( M = 6 \) sensors, with \( M_1 = M_2 = 3 \) and \( d_1 = \lambda_0/2 \). Additive noise in antenna elements is assumed as spatially and temporally independent complex Gaussian noise with zero mean and unit variance. The uncertainty angular sector of the desired signal is assumed to be \( \theta = [\theta_p - 5, \theta_p + 5] \), where \( \theta_p \) is the presumed direction towards the desired signal.

Fig. 2 compares the beampatterns of our proposed beamformer and MVDR beamformer \[1\] in presence of desired signal DOA mismatch. We assume \( \theta_p = 3^\circ \) and \( \theta_i = 5^\circ \), where \( \theta_i \) is the true DOA of the desired signal. Seven interferences are assumed to impinge on the 6-element array from directions \(-60^\circ, -45^\circ, -30^\circ, -20^\circ, 25^\circ, 40^\circ, 55^\circ \) with interference-to-noise ratio (INR) 30 dB. The SNR is 30 dB and the number of snapshots is \( Q = 30 \). The vertical dotted lines in this figure denote the DOAs of the interferences as well as the desired signal. The horizontal dotted line in this figure corresponds to 0 dB. It can be clearly seen from Fig. 2 that all of the 7 interferences are suppressed by both methods, but the proposed beamformer put deeper nulls at the DOAs of interferences than those of the MVDR beamformer. Meanwhile, the proposed beamformer maintain distortionless response at the actual DOA of desired signal, while the MVDR beamformer erroneously put deep nulls as a result of signal self-nulling.

![Fig. 2. Comparison of the beampatterns of MVDR beamformer and proposed beamformer.](image-url)

![Fig. 3. Coherent local scattering case. (a) Output SINR versus Input SNR. (b) Output SINR versus number of snapshots.](image-url)
phenomenon. Thus, the proposed beamformer has better performance compared with the MVDR beamformer, and nested array can suppress more interferences than the number of actual physical sensors.

Fig. 3 depicts the comparison results of the proposed beamformer with the following five methods in terms of the output signal-to-interference-plus-noise ratio (SINR): (i) the diagonally loaded sample matrix inversion (LSMI) beamformer [3]; (ii) the eigenspace beamformer [5]; (iii) the worst-case beamformer [6]; (iv) the SQP beamformer [12]; and (v) the beamformer of [14]. In this example, the desired signal steering vector is distorted by coherent local scattering effects and can be modeled as $\mathbf{b}_1 = \mathbf{b}_1 + \sum_{k=1}^{n} e^{j\phi_k} \mathbf{b}_k(\theta_k)$, where $\mathbf{b}_1$ is the presumed desired signal steering vector corresponding to direction $5$ and $\mathbf{b}_k(\theta_k)(k = 1, \ldots, 4)$ corresponds to scattered paths. The angles $\theta_k(k = 1, \ldots, 4)$ are randomly and independently drawn in each simulation run from a uniform generator with mean $5^\circ$ and standard deviation $2^\circ$. The path phases $\phi_k(k = 1, \ldots, 4)$ are independently and uniformly taken from the interval $[0, 2\pi]$ in each simulation run. Three interferences are assumed to impinge on the antenna array from directions $\{-20^\circ, 35^\circ, 50^\circ\}$ with

\[
\text{INR} = 30 \text{ dB}. \quad \text{Diagonal loading factor of the LSMI beamformer is selected as twice the noise power as recommended in [3].}
\]

The value $\epsilon = 0.3\text{N}$ is used for the worst-case beamformer as recommended in [6]. Fig. 3(a) displays the mean output SINR of the six methods versus the input SNR for fixed training sample size $Q = 50$. Fig. 3(b) displays the mean output SINR of the same methods versus the number of training snapshots for the fixed input SNR $= 20$ dB. For obtaining each point in the simulation examples, 100 independent Monte Carlo runs are used. It can be noted from Fig. 3 that the proposed beamformer outperforms the other algorithms and is close to the optimum SINR. This is due to the fact that the proposed method can reconstruct the interference-plus-noise covariance matrix and estimate the true desired signal steering vector with a higher accuracy than other approaches. Specifically, as compared to the recently proposed beamformer of [14], the beamforming method described in this communication has a significant performance improvement, which can be attributed to the elimination of the unwanted desired signal component from the received samples and the replacement of output power maximization criterion with beamformer sensitivity minimization criterion. One can also recall the aforementioned sections to lend support to the superiority of making such an adjustment in our proposed method.

In Fig. 4, a scenario with imprecise antenna array geometry is considered. Specifically, each sensor is assumed to be randomly displaced from its original location and the displacement is drawn uniformly from the interval $[-0.03, 0.03]$ measured in wavelength. In addition to this, all other parameters are chosen as before.

Fig. 4(a) and (b) demonstrate the output SINR performance of the aforementioned techniques versus the input SNR for fixed training data size $T = 50$ and versus the number of training snapshots for fixed SNR $= 20$ dB. Similar to the previous example, the proposed method enjoys much better beamformer performance than other algorithms in a large SNR and snapshot region.

5. Conclusion

A robust beamforming method based on interference-plus-noise covariance matrix reconstruction and desired signal steering vector estimation has been proposed for nested arrays. The interference-plus-noise covariance matrix can be reconstructed by projecting the original spatially smoothed matrix into the interference subspace. The true desired signal steering vector can be precisely estimated by solving an optimization problem, which is based on minimizing the beamformer sensitivity and constraining the convergence region of the estimated steering vector. Moreover, we develop a Lagrange multiplier methodology to solve the proposed optimization problem, and the computational complexity is comparable to that of the SCB. Numerical simulation results show that the proposed beamformer utilizes the extended DOF provided by nested array efficiently and has a superior performance compared with state-of-the-art methods.

Although implementing the proposed method achieves a satisfactory performance in a very large range of SNR or snapshot number, one limitation of this approach is that the effectiveness of it is based on far-field assumption,
then the corresponding response to near-field signals may be unacceptable. In future work, we will consider extending our algorithm to the near-field case. Investigating the application of these ideas in the case of wideband signals is another interesting topic for future work.

Acknowledgments

This research was supported in part by the National Natural Science Foundation of China under Grant 61231027, 61271292 and 61431016.

The authors wish to thank the anonymous reviewers for their helpful comments.

Appendix A. Supporting information

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.sigpro.2015.02.027.

References


