Signal Matching Wavelet for Ultrasonic Flaw Detection in High Background Noise

Guangming Shi, Senior Member, IEEE, Xuyang Chen, Student Member, IEEE, Xiaoxia Song, Fei Qi, Member, IEEE, Ailing Ding

Abstract—Wavelet transform (WT) is widely applied in ultrasonic flaw detection (UFD) system due to its property of multiresolution time-frequency analysis. Those traditional WT-based methods for UFD use wavelet basis with limited types to match various echo signals (called wavelet-matching-signal), so it is difficult for those methods to achieve the optimal match between echo signal and wavelet basis. This results in limited detection ability in high background noise for those WT-based methods. In this paper, we propose a method of signal-matching-wavelet (SMW) for UFD to solve this problem. Unlike the traditional UFD system, in the proposed SMW the transmitted signal is designed to be a wavelet function for matching a wavelet basis. This makes it possible to obtain the optimal match between the echo signal and the wavelet basis. To achieve the optimal match from the aspect of energy, we derive three rules for designing transmitted signal and selecting wavelet basis. Further, the parameter selection in applying the proposed SMW to a practical UFD system is analyzed. In addition, a low-rate DWT structure is designed to decrease the hardware cost, which facilitates the practical application of the proposed SMW. The simulation results show that the proposed SMW can efficiently detect the flaw in high background noise even with SNR being lower than -20 dB, outperforming the existing methods by 5dB.

Index Terms—Energy match, high background noise, signal matching wavelet, ultrasonic flaw detection, wavelet transform

I. INTRODUCTION

ULTRASONIC flaw detection (UFD) in high background noise is in great demands in non-destructive evaluation of many industry applications, such as testing of aeronautical materials, petroleum pipeline and automotive engine. The key problem in these applications is to suppress the high background noise and separate the weak clean echo from the noise. In the 1990s, many researchers used split spectrum processing (SSP) [1]-[3] to suppress the noise for UFD. However, the SSP does not possess the multiresolution analysis property and thus is not suitable to deal with the non-stationary ultrasonic echo signal in high background noise. The wavelet transform (WT), as a multiresolution time-frequency analysis tool [4], is widely used to suppress the noise and detect the flaw echo [5-16].

Continuous WT (CWT) mainly provides a theoretical direction for UFD applications [5]-[8], while the discrete WT (DWT) is feasible for practical UFD system due to its fast calculation and thus is more preferred [9]-[16]. Those DWT-based methods for noise suppression in UFD are mainly divided into two categories. The first category applies thresholding scheme [9]-[12] in which only the coefficients larger than a threshold are preserved, and then the signal is reconstructed with the preserved coefficients. The second category is pruning-based methods, which cut the coefficients of the un-interesting scales (regarded as noise) and preserve those of interesting scales in WT domain. [13]-[16]. Besides, to improve the detecting performance in noise environment, some researchers proposed matching-wavelet-based methods [17]-[19]. In general, those existing WT-based methods [5-19] can be generalized to use a wavelet basis to match the echo of a transmitted signal, called wavelet-matching-signal. However, under high background noise, the existing wavelet-based methods cannot solve the UFD problem efficiently. This is because, those methods do not take into account the match between the echo signal and wavelet basis during the construction of transmitted signal. It will result in a problem of using limited types of wavelet basis to match various echo signals in the processing of echo signals. This makes it difficult to achieve the optimal match and thus the detection performance for weak signal in high background noise will not be satisfactory for those industry applications mentioned above. To our knowledge, the acknowledged best result comes up when the input signal-to-noise-ratio (SNR) of echo signal reaches -15dB [6].

To address the UFD problem under high background noise, this paper proposes a method of signal-matching-wavelet (SMW). Different from those methods of wavelet-matching-signal, the proposed SMW design the transmitted signal to be a wavelet function. The idea of SMW is feasible according to the fact that an arbitrary transmitted signal can be designed by a controllable transducer [20, 21].
In this way, it is possible to achieve the optimal match between the echo signal and wavelet basis and the detection performance could be further improved.

Here, we study SMW from the perspective of optimal energy match due to the fact that the clean echo and the noise have different energy distributions in WT domain. To achieve the optimal energy match, we need to solve two problems, the concentration of flaw echo and the separation of flaw echo from noise in WT domain. By analyzing the two problems, we derive three match rules for designing transmitted signal and selecting wavelet basis. Based on the match rules and the actual requirements in UFD, we analyze the selection of parameters used in the proposed method. The above rules and requirements may need a multi-level wavelet decomposition to get better performance of noise suppression, while it will lead to a high sampling rate and difficult implementation in hardware. Thus, we design a polyphase-decomposition-based low-rate DWT structure to decrease the hardware cost and further facilitate the practical application. Simulation results illustrate that the proposed SMW can efficiently detect the flaw in high background noise even with SNR being lower than -20 dB, outperforming the existing methods by 5dB.

The rest of this paper is organized as follows. Section II gives the analysis of echo signal on energy distribution. Section III proposes the signal matching wavelet. Section IV gives the design of SMW for UFD. Experimental results in Section V manifest excellent performances of UFD. We close in Section VI with conclusions.

II. PROCEDURE FOR ANALYSIS OF ECHO SIGNAL ON ENERGY DISTRIBUTION

A. Echo Signal Model

In UFD system, the echo signal contains the flaw echo and the background noise, and the flaw information can be obtained by suppressing the noise and detecting the echo signal. Next, we introduce briefly the echo signal model.

Let \( x(t) \in L^2(\mathbb{R}) \) be the echo signal, \( r(t) \) be the clean echo signal (namely flaw echo), and \( n(t) \) be the background noise with Gaussian distribution, which is generated by a randomly distributed scatterers in the diagnosed material. Then the echo signal model is established as follows

\[
x(t) = r(t) + n(t),
\]

In this paper, we study the flaw detection of metal material by A-scan UFD system and suppose that the ultrasonic trace is frequency-independent, homogeneous and non-dispersive in the metal material. So, the clean echo signal \( r(t) \) can be expressed by

\[
r(t) = \sum_{i} \alpha_{i} s(t - \tau_{i}), i \geq 1,
\]

where \( s(t) \) is the transmitted signal, \( \tau_{i} \) is the delay of the \( i \)-th flaw echo and \( \alpha_{i} s(t - \tau_{i}) \) is the \( i \)-th flaw echo. The parameter \( \alpha_{i} \) is determined by the size of the \( i \)-th flaw and the attenuation of the material. Since \( \alpha_{i} \) denotes only an amplitude of the flaw echo, we use \( s(t - \tau_{i}) \) to represent the \( i \)-th flaw echo in the following discussion.

B. Analysis of Echo Signal on Energy Distribution in Wavelet Domain

Since the energy distribution of clean echo signal is different from that of noise in WT domain, it is necessary to analyze the energy distribution of an echo signal for noise suppression. DWT is an efficient tool for wavelet analysis which can be performed by a dyadic tree structure. By using DWT decomposition, an echo signal \( x(t) \) can be represented by

\[
x(t) = \sum_{j,k}^{\infty} d_{j,k} \cdot \psi_{j,k}(t) + \sum_{k}^{N_{max}} a_{n,k} \cdot \phi_{n,k}(t),
\]

where \( j \) and \( k \) denote the level of scale and shift amount in DWT, respectively, and \( \psi_{j,k}(t) \) and \( \phi_{n,k}(t) \) are the wavelet function and scale function of DWT, respectively. \( L_{max} \) is the maximal decomposition scale, \( d_{j,k}^{x} \) is the wavelet coefficients of \( x(t) \), \( a_{n,k} \) is the approximation coefficient of \( x(t) \) at scale \( L_{max} \). \( L_{max} \) should be selected large enough so that the coefficients \( \{d_{j,k}^{x}\} \) contain most information of \( r(t) \). The coefficients \( \{a_{n,k}\} \) usually contain some low-frequency interference and thus are removed to suppress the noise.

According to (1), \( d_{j,k}^{r} \) can be further decomposed by

\[
d_{j,k}^{r} = d_{j,k}^{n} + d_{j,k}^{r},
\]

where \( d_{j,k}^{r} \) and \( d_{j,k}^{n} \) represent the wavelet coefficients of \( r(t) \) and \( n(t) \) at the \( j \)-th scale, respectively. The energy of the clean echo signal and the noise in WT domain is determined by the wavelet coefficients \( d_{j,k}^{r} \) and \( d_{j,k}^{n} \). So, the analysis on the energy distribution of the echo signal is important to the flaw detection.

The existing methods, using a wavelet basis to match a signal, easily make the energy of \( d_{j,k}^{r} \) and \( d_{j,k}^{n} \) overlap under high background noise, which is shown in Fig. 1 [6]. The energy of the flaw echo locates in a region ‘A’, while the energy of the background noise is mainly distributed in a ‘long-narrow’ region ‘B’ at the lower scales in Fig. 1. Since the regions ‘A’ and ‘B’ overlap in some region, it is very difficult to separate the clean echo from the noise.
III. THE PROPOSED SIGNAL MATCHING WAVELET

A. Idea of Signal Matching Wavelet

The distribution of clean echo signal in WT domain is related to both the echo signal and the wavelet basis, while the traditional UFD methods only focus on how to select the wavelet basis to deal with an echo signal. In other words, they design or select wavelet basis to match an echo signal, which makes inevitable confusion of clean echo and noise in energy distribution, and undermines the ability of noise suppression and flaw detection in high background noise. To avoid the confusion of clean echo and noise, it is important to intentionally control the clean echo, besides the selection of wavelet basis. In view of this point, we propose a method of SMW for UFD in high background noise.

Unlike the traditional WT-based UFD detection methods, the transmitted signal in SMW is designed controllably in order that the echo signal matches properly a wavelet basis in energy distribution. By designing the transmitted signal to match a wavelet basis, we can indirectly control the energy distribution of echo signal in WT domain. To avoid the confusion of clean echo and noise, it is important to intentionally control the clean echo, besides the selection of wavelet basis. In view of this point, we propose a method of SMW for UFD in high background noise.

Fig. 1. Energy distribution of an echo signal in WT domain (overlap case).

Considering the time-frequency location property of a wavelet basis function, we directly design the transmitted signal to be a wavelet basis function (here, we focus on the orthogonal wavelet basis). Since a flaw echo signal is a delayed form of the transmitted signal as shown in (2), the flaw echo will also possess a localized distribution in WT domain. Then the transmitted signal \( s(t) \) can be expressed by the following form

\[
s(t) = W(t),
\]

where \( W(t) \) is a mother wavelet function. When a wavelet function is selected to be an actual transmitted signal, the pulse duration (that is, support length) of the signal must be determined. Considering that the support length is unstable to different wavelet functions at the same decomposition scale, which is not beneficial to our following study, we use the delay interval (denoted by \( t_s \)) of the adjacent orthogonal wavelet at the 0-th scale to indirectly describe the support length. The waveform of the wavelet function and the parameter \( t_s \) together determine a unique transmitted signal.

For the wavelet basis \( \{\psi_{j,k}(t), j, k \in \mathbb{Z}\} \) used in a DWT structure, each basis function in \( \{\psi_{j,k}(t)\} \) has its practical support length. Similar to \( t_s \), we define \( t_p \) as the time interval between \( \psi_{0,k}(t) \) and \( \psi_{0,k+1}(t) \). Based on the principle of DWT, the parameter \( t_p \) is equal to the sampling interval of an input signal in the DWT structure. According to the above definition, the basis function \( \psi_{j,k}(t) \) can be expressed as

\[
\psi_{j,k}(t) = 2^{-j} \psi(2^{-j} t - k \cdot t_p), \quad j, k \in \mathbb{Z}.
\]

Note, we use ‘wavelet basis’ to represent the basis used in a DWT system without extra explanation in the rest of the paper.

Now, we analyze the optimal energy match between the flaw echo and wavelet basis for designing the transmitted signal and selecting the wavelet basis.

As mentioned above, Fig. 1 shows an overlap case of the energy distribution of echo signal in WT domain. From Fig. 1, the energy distribution of the flaw echo has a bad localization property and overlap with the noise. In this case, the flaw echo is difficult to be separated from the noise. Different from Fig. 1, we show a non-overlap energy distribution in Fig. 2 [6], in which the energy distribution of the flaw echo is localized and far from that of the noise. In this case, the flaw echo can be easily separated from the noise. By comparing these two cases shown in Fig. 1 and Fig. 2, we derive two problems for the optimal energy match as follows:

(i) Concentration: the flaw echo signal should have a localized energy distribution.

(ii) Separation: the energy distribution area of the flaw echo should be far from that of the noise.

B. Optimal Energy Match

As mentioned above, the two problems, concentration and separation, are crucial for optimal energy match. In this subsection, we propose two concentration rules and one
separation rule to solve the two problems. We firstly present these three rules and then give their explanations.

**Concentration Rule 1:** The flaw echo $s(t - \tau_i)$ has a concentrated energy distribution around a scale $j_c$, if the following constraint is satisfied

$$t_b = 2^{-j_c} t_s, j_c \in \{1, 2, \cdots\},$$  \hspace{1cm} (6)$$

where $j_c$ is defined as the central scale where the major energy of a flaw echo $s(t - \tau_i)$ locates in WT domain.

**Concentration Rule 2:** The flaw echo $s(t - \tau_i)$ decays fast, if the conditions in the following two cases are satisfied.

(i) In the case that the scale $j$ decreases with $j < j_c$, the transmitted signal $s(t)$ has a high Lipschitz regularity, and the selected wavelet bases $\{\psi_{j,k}(t), j, k \in \mathbb{Z}\}$ have high vanishing moment and short support length.

(ii) In the case that the scale $j$ increases with $j > j_c$, the transmitted signal $s(t)$ has short support length.

**Separation Rule:** The energy distribution of a flaw echo $s(t - \tau_i)$ is far from that of background noise, if the central scale $j_c$ is selected large enough.

Next, the explanations of these rules are listed as follows.

As for Concentration Rule 1, considering the spectrum of wavelet, the transmitted signal $s(t)$ as well as the corresponding flaw echo $s(t - \tau_i)$ have a band-limited energy distribution in region $\pm \frac{1}{t_s} \left[\pi - \delta_1, 2\pi + \delta_1\right]$, where both $\delta_1$ and $\delta_2$ describe the fluctuation of the spectrum region where the major energy localizes and their values are small. Also, a wavelet basis $\{\psi_{j,k}(t)\}$ at the $j_c$-th scale has a band-limited energy distribution in $\pm \frac{1}{t_b} \left[\pi - \delta_1, 2\pi + \delta_2\right]$. Provided the constraint in (6) is satisfied, the wavelet basis function $\psi_{j_c,k}(t)$ has the similar band limitation with the transmitted signal $s(t)$. Thus the flaw echo $s(t - \tau_i)$ will exhibit a better concentrated energy distribution around scale $j_c$.

Concentration Rule 2 is given with the consideration of wavelet characteristics and its proof is listed detailedly in Appendix for the complicated derivation. The explanation of Separation Rule is straightforward from the fact: 1) the Gaussian noise has localized energy at lower scales and its energy decreases quickly with the increasing scale, 2) the flaw echo has concentrated energy distribution around scale $j_c$.

As a result, the concentration and separation are achieved by the three rules. According to these rules, the transmitted signal $s(t)$ and the wavelet bases $\{\psi_{j,k}(t)\}$ can be properly designed (or selected) to achieve the optimal match between the flaw echo and the wavelet basis.

Notably, there is a conflict between the high vanishing moment and the short support length in the construction of wavelet. Thus, during constructing practically the transmitted signal and wavelet basis, we should carefully select the parameters to solve the conflict. In the next section, we will apply the proposed SMW to the practical UFD and give the selection of parameters.

**IV. DESIGN OF PROPOSED SMW FOR UFD**

In this section, we apply the proposed SMW to UFD. First of all, we give the framework of the SMW-based UFD system. Then we analyze the selection of parameters used in the UFD system. Finally, a low-rate DWT structure is presented to facilitate the implementation of actual system.

**A. UFD Based on the Proposed SMW**

Fig. 3 shows the SMW-based UFD system. Unlike the traditional UFD system in which the design of transmitted signal is independent of the selection of wavelet basis, the proposed system adds a decision block to control the design of transmitted signal and the selection of wavelet basis, aiming to achieve the optimal match between flaw echo and wavelet basis. The echo signal obtained by a receiver is processed by the WT-based signal processing unit, which is determined by a selected wavelet basis according to the decision block. Consequently, the flaw echo is preserved and the background noise is suppressed effectively. Finally, the position and size of
the flaws can be determined via a flaw detector.

In the proposed system, the decision block is crucial for the noise suppression and flaw detection. Next, we give the decision procedure as follows:

**Step 1:** Determine the wavelet type of the transmitted signal and select the wavelet basis according to Concentration Rules.

**Step 2:** Set the parameter of transmitted signal \( t_c \) and the system parameters including central scale \( j_c \), threshold scales \( L_{\text{low}} \) and \( L_{\text{high}} \), theoretical sampling rate \( R_t \), and practical sampling rate \( R \) shown in Sec. IV-B and Sec. IV-C.

**Step 3:** Calculate the excitation signal \( e(t) \) by the method shown in [20], [21] and generate the transmitted signal to detect a material.

**Step 4:** Receive the echo signal and sample it at the rate \( R \). The sampled signal is decomposed by a low-rate DWT structure shown in Sec. IV-C, generating the wavelet coefficients. By using the pruning technique associated with the parameters in Step 2, the useful wavelet coefficients are preserved and then the denoised echo signal is reconstructed.

The above procedure includes several parameters related to the transmitted signal and the UFD system. In the following subsection, we describe explicitly the selection of these parameters for enhancing the match capability.

### B. Selection of Parameters

In this subsection, we discuss the selection of parameters, which is closely related to the performance of the proposed SMW for UFD. The parameters: the central scale \( j_c \) and the threshold scales \( L_{\text{low}} \) and \( L_{\text{high}} \) determine the performance of noise suppression. The parameter \( t_c \) and the sampling rate \( R_t \) are related to the system characteristics.

**Determination of Parameters \( L_{\text{low}} \) and \( L_{\text{high}} \) in ‘Pruning’**

The pruning technique is a simple and useful WT-based method for noise suppression in flaw detection [14]. In this paper we employ the pruning to separate the clean echo from noise. In the pruning technique, the input signal is supposed to have concentrated energy at the scales \( \{ L_{\text{low}}, L_{\text{low}} + 1, \cdots, L_{\text{high}} \} \) (\( 0 \leq L_{\text{low}} \leq L_{\text{high}} \leq L_{\text{max}} \)) apart from the noise in WT domain, where \( L_{\text{low}} \) and \( L_{\text{high}} \) are respectively the minimal and maximal threshold scales. So the recovered signal can be obtained only by the reconstruction of the coefficients in the preserved scales from \( L_{\text{low}} \) to \( L_{\text{high}} \).

When Concentration Rule 1 is satisfied, the major energy of flaw echo \( s(t - \tau_c) \) will be localized around scale \( j_c \). It can be found, for two wavelet-transform with their wavelet base \( \{ \psi_{j,k}(t) \} \) and \( \{ \bar{\psi}_{j,k}(t) \} \) satisfying

\[
\{ \bar{\psi}_{j,k}(t) = \psi_{j+Nj,k}(t), j,k, Nj \in \mathbb{Z} \}
\]

their generated wavelet coefficients \( d_{j,k} \) and \( \bar{d}_{j,k} \) satisfy the following relation

\[
\bar{d}_{j,k} = d_{j+Nj,k}, \quad Nj \in \mathbb{Z}.
\]

The relation (7) indicates that the wavelet coefficients of a flaw echo have the same energy distribution for the same wavelet base with different mother wavelets, whereas the central scale containing the major energy of the input signal is different. So we assume the central scale \( j_c \) to be fixed in the study of the energy distribution of flaw echo.

Due to the orthogonal and shift property of \( \{ \psi_{j,k}(t) \} \), the energy distribution, denoted by \( E_{j} \), of the wavelet coefficients of \( s(t - \tau_c) \) in a \( j \)-th scale is a periodic function of the variable \( \tau_c \) with a period \( 2^t b \). So it is enough to study the distribution of \( E_{j} \) in one period but not the whole wide range of \( \tau_c \). Then we denote the delay time \( \tau_c \) in one period by \( \rho_c \) and express \( E_j \) as a function of \( \rho_c \), as follows

\[
E_{j,\rho_c} = \sum_{t} |d_{j,t}|^2 = \sum_{t} \left| \int_{-\infty}^{\infty} s(t - \tau_c) \psi_{j,t}(t) dt \right|^2, \quad j \in \{0, 1, \cdots \}, \quad \rho_c \in \{0, 2^t b \}
\]

Experiments in Sec. V-A illustrate such a fact: if Concentration Rules are satisfied properly, the energy distribution \( E(j, \rho_c) \) is mainly localized at three scales: \( j_c - 1, j_c, j_c + 1 \). Based on this fact, we can select the parameters properly \( L_{\text{low}} = j_c - 1 \), and \( L_{\text{high}} = j_c + 1 \), which guarantees the major energy of flaw echo preserved.

**Determination of Central Scale \( j_c \)**

The central scale \( j_c \) influences the separation degree of flaw echo from noise and the sampling rate of the practical system according to Separation Rule and (6). To determine a proper \( j_c \), the energy distribution of the white Gaussian noise should be quantitatively analyzed in WT domain. We calculate the energy distribution of the Gaussian noise in several typical wavelets by percentage and show the results in Table 1.

Table 1 shows that the white Gaussian noise has the similar energy distribution for different wavelets. Specifically, over 96% energy of white Gaussian noise is localized at the first 5 scales, over 98% energy at the first 6 scales, and 99% energy at the first 7 scales. Under high background noise, the wavelet coefficients of at least the first 5 scales should be discarded in order to obtain a good result of noise suppression. So the central scale \( j_c \) should be chosen larger than 6, which is a lower bound of \( j_c \). A practical \( j_c \) should be determined by both the system cost and the noise intensity.
and Sampling Rate

is proportional

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can be

(MHz)

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is a linear module with

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-input and 1-output. Each element of

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is the velocity of ultrasound propagating in the metal

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which is the

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is needed to suppress the

. According to the

ultrasound velocity is

applied to the flaw detection in steel sample, where the

numerical range of

is

0.6, 5.2 mm when the steel sample has

several centimeters or decimeters in thickness [5]. Here we set

ζ = 1 and calculate the parameter

and

for the central scale

being 7, 8 and 9, respectively. Table 2 shows the ranges of the parameter

and the sampling rate

for each selected

. Table 2 shows that the sampling rate

varies higher with increasing

. For

, the upper bound of

even reaches GHz order of magnitude, which results in the high

hardware cost. In fact a larger

is needed to suppress the noise with high-intensity. In the following subsection, we will give an equivalent DWT structure to reduce the sampling rate without degrading noise suppression performance.

C. Low-rate DWT Structure Based on Polyphase Decomposition

In order to reduce the sampling rate of the DWT structure, an equivalent DWT structure is constructed based on polyphase decomposition [22]. As shown in Sec. IV-A, the wavelet coefficients only at the scales 

are preserved to reconstruct the signal. All these wavelet coefficients at the scales above can be obtained from the approximation coefficients 

, 

, so we only need study the equivalent DWT structure at the first

scales shown in Fig. 4. In Fig. 4, the matrix

is a linear module with

-input and 1-output. Each element of

can be expressed as

TABLE 1.
STATISTIC ENERGY DISTRIBUTION OF GAUSSIAN NOISE IN WT DOMAIN FOR SEVERAL TYPICAL WAVELETS.

<table>
<thead>
<tr>
<th>Selected wavelet basis</th>
<th>Energy distribution (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>first 4</td>
</tr>
<tr>
<td>Haar</td>
<td>93.46</td>
</tr>
<tr>
<td>db4</td>
<td>93.46</td>
</tr>
<tr>
<td>db10</td>
<td>93.78</td>
</tr>
<tr>
<td>sym6</td>
<td>94.02</td>
</tr>
<tr>
<td>sym12</td>
<td>93.04</td>
</tr>
</tbody>
</table>

Determination of Parameter

and Sampling Rate

Both the parameter

and the system sampling rate

are related to the required detection precision

which is the minimal distance between two flaws distinguished. Here we will derive their relations.

Let

be the distance propagated by an ultrasound in the time interval

in a certain metal material. The relation between

and

can be expressed as

, (9)

where

is the velocity of ultrasound propagating in the metal medium, the distance

and the detection precision

are related though a calibration parameter

. Usually different wavelet corresponds to a different

. From (9), it can be seen that the parameter

is proportional to

. It indicates that a high detection precision can be achieved by setting a small

which indicates a short support length of transmitted signal.

As is well known, the sampling rate

of the system is inversely proportional to the sampling interval

. And considering

, (6), we express

as

Combining (9), we can deduce the sampling rate

by

, (10)

Here, we give a practical example to visually show the numerical range of

and

. We assume that the UFD is applied to the flaw detection in steel sample, where the ultrasound velocity is

. According to the practical requirement, the detection precision

should belong to the region [0.6, 5.2] mm when the steel sample has

several centimeters or decimeters in thickness [5]. Here we set

ζ = 1 and calculate the parameter

and

for the central scale

being 7, 8 and 9, respectively. Table 2 shows the ranges of the parameter

and the sampling rate

for each selected

.

TABLE 2.
RANGES OF

AND

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<table>
<thead>
<tr>
<th>P (mm)</th>
<th>t_s (μs)</th>
<th>j_c = 7</th>
<th>j_c = 8</th>
<th>j_c = 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>0.38</td>
<td>39.38</td>
<td>78.77</td>
<td>157.54</td>
</tr>
<tr>
<td>5.2</td>
<td>3.26</td>
<td>341.33</td>
<td>682.67</td>
<td>1365.33</td>
</tr>
</tbody>
</table>
transferring of the delayed versions of $a_{0,k}$. We find there are two sampling operations when the analog echo signal is transformed in DWT. In order to reduce the sampling rate for analogy echo signal, we could merge these two sampling procedure. Based on this consideration, we design a low-rate equivalent DWT structure, shown in Fig. 5.

In Fig. 5, the paralleled channel number is $M = 2Q$. Each A/D sampler works at a low sampling rate $R = R_c / M$ at different sampling time in this structure. The sampling time of each A/D sampler is controlled by a high-rate clock and its sampling delay corresponding to the first A/D sampler is marked in Fig. 5. The A/D samplers are followed by the digital time calibration module and the linear module $E_Q(z)$. Since the structure in Fig. 5 is only the equivalence of the DWT structure at the first $Q$ scales, the output $a_{Q,k}$ should be decomposed consecutively from the scale $Q+1$ to $j_c + 1$, in order to obtain the wavelet coefficients at scales $\{j_c-1, j_c, j_c + 1\}$.

With the low-rate equivalent DWT structure, the sampling rate of the detection system can be significantly reduced to $R_c / 2^Q$, which shows its superiority in practical applications. The higher $Q$ is, the lower the sampling rate. However, the higher $Q$ is, the more the parallel channels $M$ and the A/D samplers. Therefore, we should trade off $Q$ and the hardware costs in the design of a practical detection system.

V. EXPERIMENTAL RESULTS

In this section, we give the experiments from two aspects, energy match and noises suppression, to illustrate the performance of the SMW.

A. Performance of Optimal Energy Match of SMW

The performance of the optimal energy match can be determined by two problems: concentration and separation. Since the separation is only determined by the central scale and the central scale can be easily adjusted to perform the separation, we only focus on the concentration of energy. Here, we give three experiments to illustrate the ability of concentration of our proposed method. The first and the second are given to illustrate the energy distribution in the aspect of the wavelet basis and the transmitted signal, respectively. The third experiment is given to show that the echo energy will be dispersed when the Concentration Rule is not satisfied.

Experiment 1: We test the influence of wavelet basis on the energy distribution of flaw echo. Here, the wavelet bases are sym3, sym5, sym8, sym13, sym18, and sym25, respectively, the transmitted signal is selected to be sym3, and the central scale $j_c = 8$. The continuous variable $\tau$ in (8) is discretized into 256 samples, thereby obtaining 256 delay values. Fig. 6 (a)–(f) shows the energy distribution $E(j, \tau)$ of the flaw echo for each selected wavelet basis. In these figures, the horizontal-axis is the scale $j$ varying from 1 to 11 and the vertical-axis is the delay $\tau$ varying from 0 to 255. Obviously, the major energy of the flaw echo concentrated at the 7th, 8th and 9th scales.

To illustrate the energy distribution of flaw echo further, the average values of $E(j, \tau)$ of all delays at each scale are calculated and shown in Table 3. It can be seen from Table 3, with the vanishing moment of wavelets increasing, the energy of flaw echo at the 7th, 8th and 9th scales tends to increase gradually. This result illustrates that the concentration of energy distribution corresponds to Concentration Rules. But when we select sym8 to sym25 as the wavelet bases, the increasing trend of energy becomes obviously slow. The reason is that such wavelet bases usually have longer support length, which is not suitable for the energy concentration. So the balance of the vanishing moment and the support length for selecting the wavelet basis is important in a practical system.
Experiment 2: This experiment tests the effect of the transmitted signal on the energy distribution of flaw echo. Here, the transmitted signal is sym3, sym5, sym8, sym10 and sym13, respectively, and the wavelet basis is selected to be sym8. And the central scale \( j_c \) is selected to 8. From the results shown in Table 4, we can conclude that the transmitted signal with a higher Lipschitz regularity results in a more concentrated energy distribution of flaw echo, which conforms to Concentration Rule 2.

Experiment 3: We further verify the influence of Concentration Rule 1 on energy distribution of flaw echo. Here the transmitted signal is selected to be sym3 wavelet, and the wavelet basis is sym8. Parameters \( t_s \) and \( t_b \) are selected as follows:

\[
t_b = a \cdot 2^{j_f} t_s, \quad a \in [1, 2], \quad j_f \in \mathbb{Z},
\]

where \( j_f \) represents the scale including the major energy of flaw echo and \( j_f = 8 \). Only when \( a = 1 \) or 2 in the above equation, the constraint of Concentration Rule 1: \( t_b = 2^{-j_c} t_s \) is satisfied. Then we test the average distribution of \( E(j, \rho_r) \) for all the \( \tilde{\rho}_r \) with the parameter \( a \) being 1.0, 1.2, 1.4, 1.6, 1.8, 2.0, respectively. As Table 5 shows, in the case of \( 1 < a < 2 \), the energy of flaw echo is dispersed to four scales (from the 6-th to 9-th scale), which decreases the performance of noise

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**Table 3.** Average energy of \( E(j, \rho_r) \) with each wavelet basis.

<table>
<thead>
<tr>
<th>Wavelet basis</th>
<th>( j \leq 5 )</th>
<th>( j = 6 )</th>
<th>( j = 7 \sim 9 )</th>
<th>( j &gt; 9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>sym3</td>
<td>0.49</td>
<td>2.63</td>
<td>96.32</td>
<td>0.55</td>
</tr>
<tr>
<td>sym5</td>
<td>0.39</td>
<td>1.66</td>
<td>97.78</td>
<td>0.16</td>
</tr>
<tr>
<td>sym8</td>
<td>0.33</td>
<td>1.40</td>
<td>98.25</td>
<td>0.01</td>
</tr>
<tr>
<td>sym13</td>
<td>0.32</td>
<td>1.38</td>
<td>98.27</td>
<td>0.01</td>
</tr>
<tr>
<td>sym18</td>
<td>0.29</td>
<td>1.56</td>
<td>98.12</td>
<td>0.00</td>
</tr>
<tr>
<td>sym25</td>
<td>0.21</td>
<td>1.49</td>
<td>98.29</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Table 4.** Average energy of \( E(j, \rho_r) \) at the 7~9-th scales of five transmitted signals.

<table>
<thead>
<tr>
<th>Transmitted signal</th>
<th>sym3</th>
<th>sym5</th>
<th>sym8</th>
<th>sym10</th>
<th>sym13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average energy (%)</td>
<td>98.25</td>
<td>99.57</td>
<td>99.92</td>
<td>99.96</td>
<td>99.97</td>
</tr>
</tbody>
</table>
suppression. While the energy distribution is concentrated at three scales for \( a = 1 \) and \( a = 2 \).

From the above three experiments, if the transmitted signal and the wavelet basis conform to Concentration Rules, the energy distribution of the echo signal is concentrated and the optimal energy match is achieved.

**B. Performance of Noise Suppression by SMW**

In this subsection, we test the performance of noise suppression and the robustness of the proposed method. Before showing experimental results, some definitions are given as follows. The input-SNR: \( \text{SNR}_i \) (or \( \text{SNR}_{i(0)} \)) is defined as the SNR of the echo signal in the whole signal length (or in the support region of the \( i \)-th flaw echo). The output-SNR: \( \text{SNR}_O \) (or \( \text{SNR}_{O(i)} \)) is defined as the SNR of the reconstructed echo signal after noise suppression in the whole signal length (or in the support region of the \( i \)-th flaw echo).

Firstly, a concrete example of the noise suppression is given. Here, we select sym5 and sym13 as the transmitted signal and the wavelet basis, respectively. The clean echo signal \( r(t) \) includes five flaw echoes with various energy intensities, which are labeled from 1 to 5 in Fig. 7(a). The noisy echo signal is generated by adding the Gaussian white noise \( n(t) \) to the clean echo signal. Then the performance of the proposed method on noise suppression is tested in two cases below.

Case 1: the noisy echo signal (including the flaw echoes 1~5) \( \text{SNR}_i = -9.18 \text{ dB} \) and the 5th flaw echo \( \text{SNR}_{i(5)} = -15.03 \text{ dB} \) (as Fig. 7(b) shows).

Case 2: the noisy echo signal (including the flaw echoes 1~5) \( \text{SNR}_i = -15.17 \text{ dB} \) and the 5th flaw echo \( \text{SNR}_{i(5)} = -20.31 \text{ dB} \) (as Fig. 7(c) shows).

Considering the intensity of noise, we choose the central scale \( j_e \) for the two cases as 8 and 10, respectively. Fig. 7(d) and (e) show the reconstructed echo signals by the SMW method. It can be found that even the flaw echoes are almost (in

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**Table 5. Average Energy at Various Scale \( j \) and Parameter \( a \).**

<table>
<thead>
<tr>
<th>( a )</th>
<th>( j \leq 5 )</th>
<th>( j = 6 )</th>
<th>( j = 7 )</th>
<th>( j = 8 )</th>
<th>( j = 9 )</th>
<th>( j &gt; 9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.33</td>
<td>1.40</td>
<td>22.37</td>
<td>64.13</td>
<td>11.75</td>
<td>0.01</td>
</tr>
<tr>
<td>1.2</td>
<td>0.59</td>
<td>5.60</td>
<td>31.05</td>
<td>53.65</td>
<td>9.32</td>
<td>0.01</td>
</tr>
<tr>
<td>1.4</td>
<td>0.84</td>
<td>9.72</td>
<td>38.23</td>
<td>42.11</td>
<td>7.05</td>
<td>0.01</td>
</tr>
<tr>
<td>1.6</td>
<td>1.17</td>
<td>14.90</td>
<td>47.40</td>
<td>33.04</td>
<td>4.15</td>
<td>0.00</td>
</tr>
<tr>
<td>1.8</td>
<td>1.47</td>
<td>19.22</td>
<td>55.82</td>
<td>22.23</td>
<td>2.30</td>
<td>0.00</td>
</tr>
<tr>
<td>2.0</td>
<td>1.73</td>
<td>22.37</td>
<td>64.13</td>
<td>11.75</td>
<td>0.01</td>
<td>0.00</td>
</tr>
</tbody>
</table>

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![Fig. 7](image_url)
the first case) or totally (in the second case) buried in the noise, all the flaw echoes can be detected successfully. By calculation, the SNR of the output echo signal are: Case 1: $SNR_O = 9.39 \text{ dB}$, $SNR_{O(5)} = 4.90 \text{ dB}$ (as Fig. 7(d) shows) and Case 2: $SNR_O = 9.58 \text{dB}$, $SNR_{O(5)} = 5.18 \text{dB}$ (as Fig. 7(e) shows), respectively. The above experiment shows the SMW method can efficiently detect the flaw in high background noise.

In the follows, we further verify the performance of noise suppression in various noise levels. Here, the input-SNR is select in the interval $[-25, 5]$ dB. The echo signal includes one flaw echo. We select three central scales $j_c$ being 8, 9 and 10, respectively. To obtain the stable statistical result, we test 100 times for each selected central scale and in each test the delay of flaw echo is randomly set. The approximate linear relation of the output-SNR against the input-SNR is depicted in Fig. 8. The average SNR improvements reach 19.40 dB, 21.96 dB and 24.47 dB for the three cases of $j_c$, respectively.

As pointed in [6], the flaw echo can be detected when the output-SNR is higher than 4 dB. As Fig. 8 shows, for the central scales $j_c = 8, 9, 10$, the flaw echo can be detected with input-SNR above -15.40 dB, -17.96 dB and -20.47 dB, respectively. So, the SMW-based method is effective in the improvement on noise suppression.

As the experimental results show, the proposed SMW is effective in the noise suppression and the flaw detection. Compared with [6], the proposed method can improve the flaw echo detection ability about 5 dB in high background noise.

VI. CONCLUSIONS

In this paper, we propose the SMW method for UFD in high background noise. SMW overcomes the shortcoming of traditional methods which is difficult to generate the optimal match due to the constraints of wavelet. In SMW, the transmitted signal is designed to be a wavelet function to obtain a localized energy distribution of flaw echoes in WT domain. Then three rules for optimal energy matching are proposed by analyzing the energy distribution of the echo signal in WT domain. Furthermore, the scheme for choosing parameters is put forward in applying the proposed SMW to the actual UFD. In addition, a low-rate equivalent DWT structure based on polyphase decomposition is developed, which reduces the hardware cost and farther facilitates the practical application. The sufficient experiments are provided in two aspects: the validation on optimal match from energy distribution of flaw echo in WT domain and the performance of noise suppression. The experimental results show that SMW can efficiently detect the flaw under high background noise even for the input-SNR low to -20 dB.

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APPENDIX

Proof of Concentration Rule 2

To facilitate the proof, we first cite a lemma which describes the decay property of a signal \( f \) in WT domain.

**Lemma 1** [4]: If a mother wavelet \( \psi(t) \) has a vanishing moment \( K \), \( K \in \mathbb{Z}^+ \), and there is a function \( f \in L^2(\mathbb{R}) \) which is uniformly Lipschitz \( \alpha < K \) over \([\xi_1, \xi_2]\), then there exists \( A > 0 \) such that

\[
\forall(u, s) \in [\xi_1, \xi_2] \times \mathbb{R}, |Wf(u, s)| \leq A \alpha^{\alpha + 1/2}, \quad (A1)
\]

where \( u \) is the shift parameter, \( s \) is the scale parameter, and \( Wf(u, s) \) is the wavelet coefficients corresponding to the WT of \( f \) on the basis \( \left\{ \frac{1}{\sqrt{s}} \psi\left(\frac{t-u}{s}\right), (u, s) \in [\xi_1, \xi_2] \right\} \).

The proof of Concentration Rule 2 can be addressed in the following two cases. Case I: scale \( j < j_c \) and Case II: scale \( j > j_c \).

**Case I**: scale \( j < j_c \).

We give the proof from three aspects: Lipschitz regularity, vanishing moment and support length. For easing the deduction, the support region of a flaw echo \( s(t - \tau) \) is denoted by \( R_c \) and its length is denoted by \( |R_c| \).

**Lipschitz regularity:**

Let the transmitted signal \( s(t) \) be uniformly Lipschitz \( \alpha_0 \) over \([-\infty, \infty]\), then a flaw echo \( s(t - \tau) \), as a delayed version of \( s(t) \), is also Lipschitz \( \alpha_0 \). Applying the Lemma above to DWT, if a basis \( \{ \psi_{j,k}(t) \} \) has a vanishing moment \( K > \alpha_0 \), the following relation is satisfied

\[
\forall j, k \in \mathbb{Z}, \quad |d'_{j,k}| = \left| \int_{-\infty}^{\infty} s(t - \tau) \psi_{j,k}(t) dt \right| \leq C_{\alpha} (2^j)^{\alpha_0 + 1/2}, \quad (A2)
\]

It can be found from (A2) that for a basis \( \{ \psi_{j,k}(t) \} \) with vanishing moment \( K > \alpha_0 \), when the scale \( j \) decrease, the wavelet coefficients \( d'_{j,k} \) decays with the degree of \( \alpha_0 + 1/2 \). Thus, if the transmitted signal \( s(t) \) have a large uniformly Lipschitz \( \alpha_0 \), when scale \( j \) decreases (\( j \to 0 \)), the wavelet coefficients has a fast decay.

**Vanishing moment:**

Let \( \psi_{j,k}(t) \) have a vanishing moment \( K_\psi \), \( K_\psi \in \mathbb{Z} \) and its support region be \( R_{j,k} \). Let \( s(t - \tau) \) be uniformly Lipschitz \( \alpha_{j,k} \) in \( R_{j,k} \). The relation \( \alpha_{j,k} \geq \alpha_0 \) can be easily obtained.

And the smaller the length of \( R_{j,k} \), the larger \( \alpha_{j,k} \). Given an arbitrary pair \( (j_i, k_i) \) satisfying \( j_i < j_c \) and \( 2^{j_i} k_i \in R_c \), the following relation is deduced according to Lemma 1.

\[
\forall j < j_c, 2^j k \in R_{j,k}, |d_{j,k}^r| = \left| \int_{-\infty}^{\infty} s(t - \tau) \psi_{j,k}(t) dt \right| \leq C_2 (2^j)^{\beta_{j,k} + 1/2}, \quad C_2 \in \mathbb{R}
\]

where \( \beta_{j,k} = \min\{\alpha_{j,k}, K_\psi\} \). Equation (A3) shows that, if the vanishing moment \( K_\psi \) is large enough, when the scale parameter \( j \) decreases, the wavelet coefficients has the decay with the degree higher than \( \alpha_0 + 1/2 \).

**Support length:**

Given two wavelet bases, one is \( \{ \psi_{j,k}(t) \} \) with vanishing moment \( K_\psi \) and support region \( R_{j,k} \) for the function \( \psi_{j,k}(t) \), and the other is \( \{ \hat{\psi}_{j,k}(t) \} \) with the same vanishing moment \( K_\psi \) but different support region \( \hat{R}_{j,k} \) for function \( \hat{\psi}_{j,k}(t) \). Let the lengths of these two support regions satisfy

\[
|R_{j,k}| < |\hat{R}_{j,k}| \quad \text{and} \quad \hat{R}_{j,k} \subset R_{j,k}.
\]

The wavelet coefficients \( s(t - \tau) \) on \( \{ \psi_{j,k}(t) \} \) and \( \{ \hat{\psi}_{j,k}(t) \} \) are denoted by \( d_{j,k} \) and \( \hat{d}_{j,k} \), respectively. Then for an arbitrary pair \( (j_2, k_2) \) satisfying \( j_2 < j_c \) and \( 2^{j_2} k_2 \in R_c \), we have \( \hat{R}_{j_2,k_2} \subset R_{j_2,k_2} \).

Let \( s(t - \tau) \) be uniformly Lipschitz \( \alpha_{j_2,k_2} \) in the region \( R_{j_2,k_2} \) and \( \gamma_{j_2,k_2} \) in the region \( \hat{R}_{j_2,k_2} \) respectively. Then we can get \( \gamma_{j_2,k_2} \geq \alpha_{j_2,k_2} \). Let \( \chi_{j_2,k_2} = \min\{\gamma_{j_2,k_2}, K_\psi\} \), we have

\[
\forall j < j_2, 2^{j_2} k \in \hat{R}_{j_2,k_2}, |d_{j,k}^r| = \left| \int_{-\infty}^{\infty} s(t - \tau) \psi_{j,k}(t) dt \right| \leq C_3 (2^j)^{\chi_{j_2,k_2} + 1/2}, \quad C_3 \in \mathbb{R}
\]

(A4)

Considering the arbitrariness of \( (j_2, k_2) \), we conclude that for a fixed vanishing moment, the decay of wavelet coefficients is faster if the support length of selected wavelet is smaller.

Combining these three relations (A2)-(A4), the Concentration Rule 2 for Case I is concluded.

**Case II**: scale \( j > j_c \).

Since the flaw echo \( s(t - \tau) \) is selected as a wavelet function from the wavelet basis \( \{ \psi_{j,k}(t - \tau), j, k \in \mathbb{Z} \} \), it satisfies the dilation and shift relations. Thus we express the coefficients \( d'_{j,k} \) as
\[ d_{jk}^f = \int_{-\infty}^{\infty} W(t - \tau) \psi_{jk}(t) dt \]

\[ u = t / 2^j - k \int_{-\infty}^{\infty} \frac{1}{\sqrt{2^j}} W \left( \frac{u - 2^j \tau}{2^j} + 2^j k \right) \psi(u) du \]

\[ = \int_{-\infty}^{\infty} W_{j, k}(-t) \psi(u) du \quad (A5) \]

The equation (A5) shows a distinct fact, not only can the coefficient \( d_{jk}^f \) be thought to be the projection of the echo signal \( W(t - \tau) \) on basis \( \psi_{jk}(t) \), but it can be thought to be the projection of \( \psi(t) \) on the basis \( W_{j, k}(-t) \). Therefore, the analysis procedure and the conclusion for Case II are similar with that for Case I, differing only on the suitable signal being \( s(t) \) instead of \( \psi_{jk}(t) \). While a higher vanishing moment of a function usually causes longer support length, which will reduce the detection precision when it is selected as a transmitted signal, so the requirement of high vanishing moment is not included in the Concentration Rule 2 for Case II. Based on the analysis above, the Concentration Rule 2 for Case II is concluded.

References


