Signal Matching Wavelet for Ultrasonic Flaw Detection in High Background Noise

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Abstract—The wavelet transform (WT) is widely used in ultrasonic flaw detection (UFd) systems because of its property of multiresolution time-frequency analysis. Those traditional WT-based methods for UFd use a wavelet basis with limited types to match various echo signals (called wavelet matching signals), so it is difficult for those methods to achieve the optimal match between echo signal and wavelet basis. This results in limited detection ability in high background noise for those WT-based methods. In this paper, we propose a signal matching wavelet (SMW) method for UFd to solve this problem. Unlike traditional UFd systems, in the proposed SMW, the transmitted signal is designed to be a wavelet function for matching a wavelet basis. This makes it possible to obtain the optimal match between the echo signal and the wavelet basis. To achieve the optimal match from the aspect of energy, we derive three rules for designing the transmitted signal and selecting the wavelet basis. Further, the parameter selection in applying the proposed SMW method to a practical UFd system is analyzed. In addition, a low-rate discrete WT structure is designed to decrease the hardware cost, which facilitates the practical application of the proposed SMW. The simulation results show that the proposed SMW can efficiently detect flaws in high background noise even with SNR lower than $-20$ dB, outperforming the existing methods by 5 dB.

I. INTRODUCTION

ULTRASONIC flaw detection (UFd) in high background noise is in great demand for non-destructive evaluation in many industry applications, such as testing of aeronautical materials, petroleum pipelines, and automotive engines. The key problem in these applications is to suppress the high background noise and separate the weak clean echo from the noise. In the 1990s, many researchers used split-spectrum processing (SSP) [1]–[3] to suppress the noise for UFd. However, SSP does not possess the multiresolution analysis property and thus is not suitable for dealing with a non-stationary ultrasonic echo signal in high background noise. The wavelet transform (WT), as a multiresolution time-frequency analysis tool [4], is widely used to suppress the noise and detect the flaw echo [5]–[16].

Continuous wavelet transform (CWT) mainly provides a theoretical direction for UFd applications [5]–[8], whereas the discrete wavelet transform (DWT) is feasible for practical UFd systems because of its fast calculation time, and thus is preferable [9]–[16]. Those DWT-based methods for noise suppression in UFd are mainly divided into two categories. The first category applies a thresholding scheme [9]–[12] in which only the coefficients larger than a threshold are preserved, and then the signal is reconstructed with the preserved coefficients. The second category uses pruning-based methods, which cut the coefficients of the uninteresting scales (regarded as noise) and preserve those of interesting scales in the WT domain [13]–[16]. Additionally, to improve the detection performance in noise environment, some researchers proposed matching-wavelet-based methods [17]–[19]. In general, those existing WT-based methods [5]–[19] can be generalized to use a wavelet basis to match the echo of a transmitted signal, called a wavelet matching signal. However, under high background noise, those methods cannot solve the UFd problem efficiently. This is because those methods do not take into account the match between the echo signal and wavelet basis during the construction of transmitted signal. This results in a problem of using limited types of wavelet bases to match various echo signals in the processing of echo signals. This makes it difficult to achieve the optimal match and thus the detection performance for a weak signal in high background noise will not be satisfactory for the previously mentioned industry applications. To our knowledge, the acknowledged best result is achieved when the input SNR of echo signal reaches $-15$ dB [6].

To address the UFd problem under high background noise conditions, this paper proposes a signal matching wavelet (SMW) method. Different from wavelet matching signal methods, the proposed SMW designs the transmitted signal to be a wavelet function. The idea of SMW is feasible because an arbitrary transmitted signal can be designed by a controllable transducer [20], [21]. In this way, it is possible to achieve the optimal match between the echo signal and wavelet basis and the detection performance could be further improved.

Here, we study SMW from the perspective of optimal energy match because the clean echo and the noise have different energy distributions in WT domain. To achieve the optimal energy match, we need to solve two problems, the concentration of flaw echo and the separation of flaw echo from noise in WT domain. By analyzing the two problems, we derive three matching rules for designing the
transmitted signal and selecting the wavelet basis. Based on the match rules and the actual requirements in UFD, we analyze the selection of parameters used in the proposed method. These rules and requirements may need a multi-level wavelet decomposition to get better performance of noise suppression, which would lead to a high sampling rate and difficult hardware implementation. Thus, we design a polyphase-decomposition-based low-rate DWT structure to decrease the hardware cost and further facilitate the practical application. Simulation results show that the proposed SMW can efficiently detect the flaw in high background noise even with SNR lower than −20 dB, outperforming the existing methods by 5 dB.

The rest of this paper is organized as follows. Section II gives the analysis of echo signal on energy distribution. Section III proposes the signal matching wavelet. Section IV gives the design of SMW for UFD. Experimental results in Section V show the excellent performance of UFD. We close in Section VI with conclusions.

II. Analysis of the Echo Signal’s Energy Distribution

A. Echo Signal Model

In UFD systems, the echo signal contains the flaw echo and the background noise, and information about the flaw can be obtained by suppressing the noise and detecting the echo signal. Next, we briefly introduce the echo signal model.

Let \( x(t) \in L^2(\mathbb{R}) \) be the echo signal, \( r(t) \) be the clean echo signal (the flaw echo), and \( n(t) \) be the background noise with Gaussian distribution, which is generated by a randomly distributed scatterers in the diagnosed material. Then, the echo signal model is established as follows:

\[
x(t) = r(t) + n(t).
\]

In this paper, we study flaw detection in metal using an A-scan UFD system and suppose that the ultrasonic trace is frequency-independent, homogeneous, and non-dispersive in the metal material. Therefore, the clean echo signal \( r(t) \) can be expressed by

\[
r(t) = \sum_i \alpha_i s(t - \tau_i), \quad i \geq 1,
\]

where \( s(t) \) is the transmitted signal, \( \tau_i \) is the delay of the \( i \)th flaw echo, and \( \alpha_i s(t - \tau_i) \) is the \( i \)th flaw echo. The parameter \( \alpha_i \) is determined by the size of the \( i \)th flaw and the attenuation of the material. Because \( \alpha_i \) denotes only an amplitude of the flaw echo, we use \( s(t - \tau_i) \) to represent the \( i \)th flaw echo in the following discussion.

B. Analysis of the Echo Signal’s Energy Distribution in the Wavelet Domain

Because the energy distribution of the clean echo signal is different from that of the noise in the WT domain, it is necessary to analyze the energy distribution of an echo signal for noise suppression. DWT is an efficient tool for wavelet analysis which can be performed by a dyadic tree structure. By using DWT decomposition, an echo signal \( x(t) \) can be represented by

\[
x(t) = \sum_{j=1}^{L_{\text{max}}} \sum_k d_{jk}^r \cdot \psi_j^r(t) + \sum_k a_{L_{\text{max}}} \phi_{L_{\text{max}}}(t),
\]

where \( j \) and \( k \) denote the level of scale and shift amount in the DWT, respectively, and \( \psi_j^r(t) \) and \( \phi_{L_{\text{max}}}(t) \) are the wavelet function and scale function of the DWT, respectively. \( L_{\text{max}} \) is the maximal decomposition scale, \( d_{jk}^r \) is the wavelet coefficients of \( x(t) \), \( a_{L_{\text{max}}} \) is the approximation coefficient of \( x(t) \) at scale \( L_{\text{max}} \). \( L_{\text{max}} \) should be selected large enough so that the coefficients \( \{d_{jk}^r\} \) contain most of the information of \( r(t) \). The coefficients \( \{a_{L_{\text{max}}}\} \) usually contain some low-frequency interference and thus are removed to suppress the noise.

According to (1), \( d_{jk}^r \) can be further decomposed by

\[
d_{jk}^r = d_{jk}^r + d_{jk}^n,
\]

where \( d_{jk}^r \) and \( d_{jk}^n \) represent the wavelet coefficients of \( r(t) \) and \( n(t) \) at the \( j \)th scale, respectively. The energy of the clean echo signal and the noise in WT domain is determined by the wavelet coefficients \( d_{jk}^r \) and \( d_{jk}^n \). Therefore, the analysis of the energy distribution of the echo signal is important to the flaw detection.

The existing methods, using a wavelet basis to match a signal, easily make the energy of \( d_{jk}^r \) and \( d_{jk}^n \) overlap under high background noise, which is shown in Fig. 1. The energy of the flaw echo is located in region A, whereas the energy of the background noise is mainly distributed in a wide, flat region B at the lower scales in Fig. 1. Because there is an area of overlap between regions A and B, it is very difficult to separate the clean echo from the noise.

III. The Proposed Signal Matching Wavelet

A. Idea of Signal Matching Wavelet

The distribution of the clean echo signal in the WT domain is related to both the echo signal and the wavelet basis, whereas the traditional UFD methods only focus on how to select the wavelet basis to deal with an echo signal. In other words, they design or select a wavelet basis to match an echo signal, which causes inevitable confusion between the clean echo and the noise in energy distribution, and undermines the noise suppression and flaw detection in high background noise conditions. To avoid the confusion of clean echo and noise, it is important to intentionally control the clean echo, besides the selection of wavelet basis. In view of this, we propose a SMW method for UFD in high background noise.
Unlike in traditional WT-based UFD detection methods, the design of the transmitted signal in SMW is controlled so that the echo signal properly matches a wavelet basis in energy distribution. By designing the transmitted signal to match a wavelet basis, we can indirectly control the energy distribution of the echo signal in the WT domain. Because both the echo signal and wavelet basis can be controlled, the optimal match between flaw echo and wavelet basis is achievable, which will result in the elimination of confusion between the clean echo and the noise in the WT domain.

Considering the time-frequency location property of a wavelet basis function, we directly design the transmitted signal to be a wavelet basis function (here, we focus on the orthogonal wavelet basis). Because a flaw echo \( s(t - \tau_i) \) is a delayed form of the transmitted signal as shown in (2), the flaw echo will also possess a localized distribution in WT domain. Thus, the transmitted signal \( s(t) \) can be expressed in the form

\[
s(t) = W(t),
\]

where \( W(t) \) is a mother wavelet function. When a wavelet function is selected to be an actual transmitted signal, the pulse duration (that is, support length) of the signal must be determined. Considering that the support length is unstable to different wavelet functions at the same decomposition scale, which is not beneficial to our following study, we use the delay interval (denoted by \( t_s \)) of the adjacent orthogonal wavelet at the 0th scale to indirectly describe the support length. The waveform of the wavelet function and the parameter \( t_s \) together determine a unique transmitted signal.

For the wavelet basis \( \{\psi_{j,k}(t), j,k \in \mathbb{Z}\} \) used in a DWT structure, each basis function in \( \{\psi_{j,k}(t)\} \) has a practical support length. Similar to \( t_s \), we define \( t_b \) as the time interval between \( \psi_{0,k}(t) \) and \( \psi_{0,k+1}(t) \). Based on the principle of DWT, the parameter \( t_b \) is equal to the sampling interval of an input signal in the DWT structure. According to this definition, the basis function \( \psi_{j,k}(t) \) can be expressed as

\[
\psi_{j,k}(t) = 2^{-j/2} \psi(2^{-j/2} t - k \cdot t_b), \quad j,k \in \mathbb{Z},
\]

Note that we use the term wavelet basis to represent the basis used in a DWT system without extra explanation in the rest of the paper.

Now, we analyze the optimal energy match between the flaw echo and wavelet basis for designing the transmitted signal and selecting the wavelet basis.

As mentioned previously, Fig. 1 shows an overlapping case of the energy distribution of the echo signal in the WT domain. From Fig. 1, the energy distribution of the flaw echo has a bad localization property and overlaps with the noise. In this case, the flaw echo is difficult to separate from the noise. Different from Fig. 1, we show a non-overlapping energy distribution in Fig. 2 [6], in which the energy distribution of the flaw echo is localized and far from that of the noise. In this case, the flaw echo can be easily separated from the noise. By comparing the two cases shown in Fig. 1 and Fig. 2, we derive two problems for the optimal energy match as follows:

1) Concentration: the flaw echo signal should have a localized energy distribution.
2) Separation: the energy distribution area of the flaw echo should be far from that of the noise.

**B. Optimal Energy Match**

As mentioned previously, the two problems, concentration and separation, are crucial for optimal energy matching. In this subsection, we propose two concentration rules and one separation rule to solve the two problems. We first present these three rules and then give their explanations.

- Concentration rule 1: The flaw echo \( s(t - \tau_i) \) has a concentrated energy distribution around a scale \( j_c \), if the following constraint is satisfied

\[
t_b = 2^{-j_c} t_s, \quad j_c \in \{1,2,\cdots\},
\]

where \( j_c \) is defined as the central scale where the major energy of a flaw echo \( s(t - \tau_i) \) locates in WT domain.
Concentration rule 2: The flaw echo \( s(t - \tau_i) \) decays fast, if the conditions in the following two cases are satisfied.

1) In the case that the scale \( j \) decreases with \( j < j_c \), the transmitted signal \( s(t) \) has a high Lipschitz regularity, and the selected wavelet bases \( \{ \psi_{j,k}(t) \}, j,k \in \mathbb{Z} \) have high vanishing moment and short support length.

2) In the case that the scale \( j \) increases with \( j < j_c \), the transmitted signal \( s(t) \) has short support length.

Separation rule: The energy distribution of a flaw echo \( s(t - \tau_i) \) is far from that of background noise, if the central scale \( j_c \) is selected large enough.

For concentration rule 1, considering the spectrum of wavelet, the transmitted signal \( s(t) \) and the corresponding flaw echo \( s(t - \tau_i) \) have a band-limited energy distribution in the region \( \pm (1/t_s)[\pi - \delta_1, 2\pi + \delta_2] \), where both \( \delta_1 \) and \( \delta_2 \) describe the fluctuation of the spectrum region where most of the energy is located and their values are small. Also, a wavelet basis \( \{ \psi_{j,k}(t) \} \) at the \( j \)-th scale has a band-limited energy distribution in \( \pm (1/t_c)[(\pi - \delta_1)/2, (2\pi + \delta_2)/2] \). Provided the constraint in (6) is satisfied, the wavelet basis function \( \psi_{j,k}(t) \) has band limitation similar to the transmitted signal \( s(t) \). Thus, the flaw echo \( s(t - \tau_i) \) will exhibit a more concentrated energy distribution around scale \( j_c \).

Concentration rule 2 is given with the consideration of wavelet characteristics and its proof is given in detail in the Appendix for its complicated derivation. The explanation of the separation rule is straightforward given that 1) the Gaussian noise has localized energy at lower scales and its energy decreases quickly with the increasing scale \( j \), and 2) the flaw echo has concentrated energy distribution around scale \( j_c \).

As a result, the concentration and separation are achieved by the three rules. According to these rules, the transmitted signal \( s(t) \) and the wavelet bases \( \{ \psi_{j,k}(t) \} \) can be properly designed (or selected) to achieve the optimal match between the flaw echo and the wavelet basis.

Notably, there is a conflict between the high vanishing moment and the short support length in the construction of wavelet. Thus, during practical construction of the transmitted signal and wavelet basis, we should carefully select the parameters to solve the conflict. In the next section, we will apply the proposed SMW to practical UFD and give the selection of parameters.

IV. DESIGN OF THE PROPOSED SMW FOR UFD

In this section, we apply the proposed SMW to UFD. First, we give the framework of the SMW-based UFd system; then we analyze the selection of parameters used in the UFd system. Finally, a low-rate DWT structure is presented to facilitate the implementation of an actual system.

A. UFd Based on the Proposed SMW

Fig. 3 shows the SMW-based UFd system. Unlike a traditional UFd system, in which the design of transmitted signal is independent of the selection of wavelet basis, the proposed system adds a decision block to control the design of transmitted signal and the selection of wavelet basis, with a goal of achieving the optimal match between the flaw echo and the wavelet basis. The echo signal obtained by a receiver is processed by the WT-based signal processing unit, which is determined by a selected wavelet basis according to the decision block. Consequently, the flaw echo is preserved and the background noise is suppressed effectively. Finally, the position and size of the flaws can be determined via a flaw detector.

In the proposed system, the decision block is crucial for noise suppression and flaw detection. The decision procedure is as follows:

- Step 1: Determine the wavelet type of the transmitted signal and select the wavelet basis according to the concentration rules.
- Step 2: Set the parameter of transmitted signal \( t_s \) and the system parameters including central scale \( j_c \).
threshold scales $L_{\text{low}}$ and $L_{\text{high}}$, theoretical sampling rate $R_t$, and practical sampling rate $R$ shown in Sections IV-B and IV-C.

- Step 3: Calculate the excitation signal $e(t)$ by the method shown in [20] and [21], and generate the transmitted signal to detect a flaw.
- Step 4: Receive the echo signal and sample it at the rate $R$. The sampled signal is decomposed by a low-rate DWT structure as shown in Section IV-C, generating the wavelet coefficients. By using the pruning technique associated with the parameters in Step 2, the useful wavelet coefficients are preserved and the denoised echo signal is reconstructed.

This procedure includes several parameters related to the transmitted signal and the UFD system. In the following subsection, we explicitly describe the selection of these parameters to enhance the matching capability.

B. Selection of Parameters

In this subsection, we discuss the selection of parameters, which is closely related to the performance of the proposed SMW for UFD. The parameters central scale, $j_c$, and the threshold scales $L_{\text{low}}$ and $L_{\text{high}}$ determine the performance of noise suppression. The parameter $t_e$ and the sampling rate $R_i$ are related to the system characteristics.

1) Determination of Parameters $L_{\text{low}}$ and $L_{\text{high}}$ in Pruning: The pruning technique is a simple and useful WT-based method for noise suppression in flaw detection [14]. In this paper, we employ pruning to separate the clean echo from the noise. In the pruning technique, the input signal is supposed to have concentrated energy at the scales $\{L_{\text{low}}, L_{\text{low}}+1, \ldots, L_{\text{high}}\}$ ($0 < L_{\text{low}} \leq L_{\text{high}} \leq L_{\max}$) apart from the noise in WT domain, where $L_{\text{low}}$ and $L_{\text{high}}$ are the minimum and maximum threshold scales, respectively. Thus, the recovered signal can be obtained only by the reconstruction of the coefficients in the preserved scales from $L_{\text{low}}$ to $L_{\text{high}}$.

When concentration rule 1 is satisfied, most of the energy of flaw echo $s(t - \tau)$ will be localized around scale $j_c$. It can be found, for two wavelet-transform with their wavelet bases $\{\psi_{j,k}(t)\}$ and $\{\tilde{\psi}_{j,k}(t)\}$ satisfying $\tilde{\psi}_{j,k}(t) = \psi_{j+\Delta,j,k}(t)$, $j,k,\Delta j \in \mathbb{Z}$, their generated wavelet coefficients $d_{j,k}$ and $\tilde{d}_{j,k}$ satisfy the following relation:

$$\tilde{d}_{j,k} = d_{j+\Delta,j,k}, \quad \Delta j \in \mathbb{Z}$$ (7)

Eq. (7) indicates that the wavelet coefficients of a flaw echo have the same energy distribution for the same wavelet base with different mother wavelets, but the central scale containing the major energy of the input signal is different. Therefore, we assume the central scale $j_c$ to be fixed in the study of the energy distribution of the flaw echo.

Because of the orthogonal and shift property of $\{\psi_{j,k}(t)\}$, the energy distribution, denoted by $E_j$, of the wavelet coefficients of $s(t - \tau)$ in a $j$th scale is a periodic function of the variable $\tau$ with a period $2^j t_e$. Therefore, it is enough to study the distribution of $E_j$ in one period rather than the whole range of $\tau$. Then, we denote the delay time $\tau_i$ in one period by $\rho_r$ and express $E_j$ as a function of $\rho_r$ as follows:

$$E_{j,\rho_r} = \sum_k |d_{j,k}|^2 = \sum_k \left| \int_{-\infty}^{\infty} s(t - \rho_r) \tilde{\psi}_{j,k}(t) dt \right|^2,$$ (8)

Experiments in Section V-A illustrate that if the concentration rules are satisfied properly, the energy distribution $E_{j,\rho_r}$ is mainly localized at three scales: $j_c - 1, j_c, j_c + 1$. Based on this fact, we can select the parameters properly: $L_{\text{low}} = j_c - 1$, and $L_{\text{high}} = j_c + 1$, which guarantees that most of the energy of the flaw echo is preserved.

2) Determination of Central Scale $j_c$: The central scale $j_c$ influences the separation degree of the flaw echo from the noise and the sampling rate of the practical system according to the separation rule and (6). To determine a proper $j_c$, the energy distribution of the white Gaussian noise should be quantitatively analyzed in the WT domain. We calculate the energy distribution of the Gaussian noise in several typical wavelets by percentage and show the results in Table I.

Table I shows that the white Gaussian noise has similar energy distribution for different wavelets. Specifically,
more than 96% energy of white Gaussian noise is localized in the first 5 scales, more than 98% energy in the first 6 scales and more than 99% energy in the first 7 scales. Under high background noise conditions, the wavelet coefficients of at least the first 5 scales should be discarded to obtain good noise suppression. Thus the central scale $j_c$ should be chosen larger than 6, which is a lower bound of $j_c$. A practical $j_c$ should be determined by both the system cost and the noise intensity.

3) Determination of Parameter $t_s$ and Sampling Rate $R_i$: Both the parameter $t_s$ and the system sampling rate $R_i$ are related to the required detection precision $P$, which is the minimum distance between two flaws that allows them to be distinguished from a single flaw. Here we will derive their relations.

Let $D$ be the distance propagated by an ultrasound wave in the time interval $t_s$ in a certain metal material. The relation between $D$ and $t_s$ can be expressed as

$$t_s = 2D/v = 2ζP/v,$$  \(9\)

where $v$ is the velocity of ultrasound propagating in the metal medium; the distance $D$ and the detection precision $P$ are related though a calibration parameter $ζ \in (0, 1]$. Usually a different wavelet corresponds to a different $ζ$. From (9), it can be seen that the parameter $t_s$ is proportional to $P$. It indicates that a high detection precision can be achieved by setting a small $t_s$, which indicates a short support length of the transmitted signal.

As is well known, the sampling rate $R_i$ of the system is inversely proportional to the sampling interval $t_b$. And considering $t_b = 2^{−j_c}t_s$ in (6), we express $R_i$ as

$$R_i = 1/t_b = 2^{j_c}/t_s.$$  \(10\)

Combining this with (9), we can deduce the sampling rate $R_i$ from

$$R_i = 2^{j_c−1}v/(ζP).$$  \(11\)

Here, we give a practical example to visually show the numerical range of $t_s$ and $R_i$. We assume that the UFD is applied to flaw detection in a steel sample, in which the ultrasound velocity is $v = 3200$ m/s. According to the practical requirement, the detection precision $P$ should belong to the region $[0.6, 5.2]$ mm when the steel sample is several centimeters or decimeters in thickness [5]. Here we set $ζ = 1$ and calculate the parameter $t_s$ and $R_i$ for central scales $j_c$ of 7, 8, and 9. Table II shows the ranges of the parameter $t_s$ and the sampling rate $R_i$ for each selected $j_c$.

Table II shows that the sampling rate $R_i$ increases with increasing $j_c$. For $j_c = 9$, the upper bound of $R_i$ even reaches the gigahertz order of magnitude, which results in high hardware cost. In fact, a larger $j_c$ is needed to suppress high-intensity noise. In the following subsection, we will give an equivalent DWT structure to reduce the sampling rate without degrading noise suppression performance.

C. Low-Rate DWT Structure Based on Polyphase Decomposition

To reduce the sampling rate of the DWT structure, an equivalent DWT structure is constructed based on polyphase decomposition [22]. As shown in Section IV-A, only the wavelet coefficients at the scales $\{j_c−1, j_c, j_c + 1\}$ are preserved to reconstruct the signal. All of these wavelet coefficients can be obtained from the approximation coefficients $\{a_{Qj_c} k \in Z\}, 1 \leq Q \leq j_c − 2$, so we only need to study the equivalent DWT structure at the first $Q$ scales shown in Fig. 4. In Fig. 4, the matrix $E_Q(z)$ is a linear module with $2^Q$-input and 1-output. Each element of $E_Q(z)$ can be expressed as

$$E_Q(z) = \sum_{n} h'(2^Q n + \hat{j})z^{−n}, \quad i = 0, 1, \ldots, 2^Q − 1,$$  \(11\)

where $h'(n) = h(n) \ast (h(n) \uparrow 2) \ast (h(n) \uparrow 2^2) \ast \cdots \ast (h(n) \uparrow 2^{Q−1})$ and $h(n)$ is the scale filter in DWT, where $\uparrow$ denotes the up-sampling of a digital signal.

In Fig. 4, we assume the input $a_{0,j_c}$ is the high-rate sampling result of an analog echo signal. It can be seen that

<table>
<thead>
<tr>
<th>Selected wavelet basis</th>
<th>Energy distribution (%)</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>First 4 scales</td>
</tr>
<tr>
<td>Haar</td>
<td>93.46</td>
</tr>
<tr>
<td>db4</td>
<td>93.46</td>
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<td>db10</td>
<td>93.78</td>
</tr>
<tr>
<td>dmev</td>
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</tr>
<tr>
<td>sym6</td>
<td>93.04</td>
</tr>
<tr>
<td>sym12</td>
<td>93.55</td>
</tr>
</tbody>
</table>

TABLE II. RANGES OF $P$, $t_s$, AND $R_i$ FOR EACH SELECTED $j_c$.
the output signal $a_{Q,k}$ is obtained by downsampling and linearly transforming the delayed versions of $a_{Q,k}$. We find that there are two sampling operations when the analog echo signal is transformed in DWT. To reduce the sampling rate for an analog echo signal, we could merge these two sampling procedures. Based on this consideration, we design a low-rate equivalent DWT structure as shown in Fig. 5. In Fig. 5, the number of parallel channels is $M = 2^Q$. Each A/D sampler works at a low sampling rate $R = R_i/M$ at different sampling time in this structure. The sampling time of each A/D sampler is controlled by a high-rate clock and its sampling delay corresponding to the first A/D sampler is marked in Fig. 5. The A/D samplers are followed by the digital time calibration module and the linear module $E_Q(z)$. Because the structure in Fig. 5 is only the equivalence of the DWT structure at the first $Q$ scales, the output $a_{Q,k}$ should be decomposed consecutively from the scale $Q + 1$ to $j_c + 1$, to obtain the wavelet coefficients at scales $(j_c - 1, j_c, j_c + 1)$.

With the low-rate equivalent DWT structure, the sampling rate of the detection system can be significantly reduced to $R_i/2^Q$, which shows its superiority in practical applications. The higher $Q$ is, the lower the sampling rate. However, the higher $Q$ is, the more parallel channels $M$ and A/D samplers are required. Therefore, there is a trade-off between $Q$ and the hardware costs in the design of a practical detection system.

V. EXPERIMENTAL RESULTS

In this section, we give the results of experiments from two aspects, energy matching and noise suppression, to illustrate the performance of the SMW.

A. Performance of Optimal Energy Match of SMW

The performance of the optimal energy match can be determined by two problems: concentration and separation. Because the separation is only determined by the central scale and the central scale can be easily be adjusted to perform the separation, we only focus on the concentration of energy. Here, we give three experiments to illustrate our proposed method’s ability to concentrate the energy. The first and the second are given to illustrate the energy distribution in the aspect of the wavelet basis and the transmitted signal, respectively. The third experiment is given to show that the echo energy will be dispersed when the concentration rule is not satisfied.

1) Experiment 1: We test the influence of wavelet basis on the energy distribution of flaw echo. Here, the wavelet bases are sym3, sym5, sym8, sym13, sym18, and sym25; the transmitted signal is selected to be sym3, and the central scale $j_c = 8$. The continuous variable $\rho\tau$ in (8) is discretized into 256 samples, thereby obtaining 256 delay values. Figs. 6(a) to 6(f) show the energy distribution $E(j,\rho\tau)$ of the flaw echo for each selected wavelet basis. In these figures, the horizontal axis is the scale $j$ varying from 1 to 11 and the vertical-axis is the delay $\tau$ varying from 0 to 255. Obviously, most of the energy of the flaw echo is concentrated at the 7th, 8th, and 9th scales.

To further illustrate the energy distribution of the flaw echo, the average values of $E(j,\rho\tau)$ of all delays at each scale are calculated and shown in Table III. It can be seen from Table III, with increasing vanishing moment of wavelets, the energy of flaw echo at the 7th, 8th and 9th scales tends to increase gradually. This result illustrates that the concentration of energy distribution corresponds to the concentration rules. However, when we select sym8 to sym25 as the wavelet bases, the increasing trend of energy becomes obviously slow. The reason is that such wavelet bases usually have longer support length, which is not suitable for energy concentration. Thus, the balance of the vanishing moment and the support length for selecting the wavelet basis is important in a practical system.

2) Experiment 2: This experiment tests the effect of the transmitted signal on the energy distribution of flaw echo. Here, the transmitted signal is chosen to be sym3, sym5, sym8, sym10, and sym13, and the wavelet basis is selected to be sym8. The central scale $j_c$ is selected to be 8. From the results shown in Table IV, we can conclude
that a transmitted signal with a higher Lipschitz regular-
ity results in a more concentrated energy distribution of
the flaw echo, which conforms to concentration rule 2.

3) Experiment 3: We further verify the influence of con-
centration rule 1 on energy distribution of flaw echo. Here,
the transmitted signal is selected to be the sym3 wavelet,
and the wavelet basis is sym8. Parameters $t_b$ and $t_s$ are
selected as follows:

$$t_b = a \cdot 2^{-j' t_s}, \quad a \in [1, 2], j' \in \mathbb{Z}^+,$$

where $j'$ represents the scale including the most energy of
the flaw echo and $j' = 8$. Only when $a = 1$ or $2$ in this
equation is the constraint of concentration rule 1, $t_b =
2^{-j' t_s}$, satisfied. Then, we test the average distribution
of $E(j, r)$ for all the $r$, with the parameter $a$ being $1.0, 1.2,
1.4, 1.6, 1.8, 2.0$. As Table V shows, in the case of $1 < a < 2$,
the energy of flaw echo is dispersed to four scales (from
the 6th to 9th scale), which decreases the performance of
noise suppression. The energy distribution is concentrated
at three scales for $a = 1$ and $a = 2$.

From these three experiments, it can be seen that if
the transmitted signal and the wavelet basis conform to
concentration rules, the energy distribution of the echo
signal is concentrated and the optimal energy match is
achieved.

B. Performance of Noise Suppression by SMW

In this subsection, we test the performance of noise
suppression and the robustness of the proposed method.
Before showing experimental results, some definitions are
given as follows. The input SNR, $\text{SNR}_I$ (or $\text{SNR}_I(r)$), is
deﬁned as the SNR of the echo signal in the whole signal
length (or in the support region of the $r$th flaw echo). The
output SNR, $\text{SNR}_O$ (or $\text{SNR}_O(r)$), is deﬁned as the SNR
of the reconstructed echo signal after noise suppression in
the whole signal length (or in the support region of the $r$th flaw echo).

First, a concrete example of the noise suppression is
given. Here, we select sym5 and sym13 as the transmitted
signal and the wavelet basis, respectively. The clean echo
signal $r(t)$ includes five flaw echoes with various energy

\begin{table}[h]
\centering
\caption{Average Energy of $E(j, r)$ With Each Wavelet Basis.}
\begin{tabular}{|c|c|c|c|}
\hline
Wavelet & Average energy (%) & \multicolumn{3}{|c|}{$j$}
\hline
\cline{3-5}
\hline
basis & $j \leq 5$ & $j = 6$ & $j = 7$ to 9 & $j > 9$
\hline
sym3 & 0.49 & 2.63 & 96.32 & 0.55
\hline
sym5 & 0.39 & 1.66 & 97.78 & 0.16
\hline
sym8 & 0.33 & 1.40 & 98.25 & 0.01
\hline
sym13 & 0.32 & 1.38 & 98.27 & 0.01
\hline
sym18 & 0.29 & 1.56 & 98.12 & 0.00
\hline
sym25 & 0.21 & 1.49 & 98.29 & 0.00
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\caption{Average Energy of $E(j, r)$ at the 7th to 9th Scales of Five Transmitted Signals.}
\begin{tabular}{|c|c|c|c|c|c|}
\hline
Transmitted signal & sym3 & sym5 & sym8 & sym10 & sym13
\hline
Average energy (%) & 98.25 & 99.57 & 99.92 & 99.96 & 99.97
\hline
\end{tabular}
\end{table}
intensities, which are labeled from 1 to 5 in Fig. 7(a).
The noisy echo signal is generated by adding the Gaussian white noise \( n(t) \) to the clean echo signal, then the performance of the proposed method on noise suppression is tested in two cases:

- Case 1: the noisy echo signal (including the flaw echoes 1 to 5) \( \text{SNR}_I = -9.18 \, \text{dB} \) and the 5th flaw echo \( \text{SNR}_{I(5)} = -15.03 \, \text{dB} \) [as shown in Fig. 7(b)].
- Case 2: the noisy echo signal (including the flaw echoes 1 to 5) \( \text{SNR}_I = -15.17 \, \text{dB} \) and the 5th flaw echo \( \text{SNR}_{I(5)} = -20.31 \, \text{dB} \) [as shown in Fig. 7(c)].

Considering the intensity of noise, we choose the central scale \( j_c \) for the two cases as 8 and 10, respectively. Figs. 7(d) and 7(e) show the reconstructed echo signals by the SMW method. It can be found that even though the flaw echoes are almost (in the first case) or totally (in the second case) buried in the noise, all of the flaw echoes can be detected successfully. By calculation, the SNRs of the output echo signal are, for Case 1, \( \text{SNR}_O = 9.39 \, \text{dB} \), \( \text{SNR}_{O(5)} = 4.90 \, \text{dB} \) [as shown in Fig. 7(d)], and for Case 2, \( \text{SNR}_O = 9.58 \, \text{dB} \), \( \text{SNR}_{O(5)} = 5.18 \, \text{dB} \) [as shown in Fig. 7(e)]. This experiment shows that the SMW method can efficiently detect a flaw in high background noise.

In the following, we further verify the performance of noise suppression for various noise levels. Here, the input SNR is selected in the interval \([-25, 5]\) dB. The echo signal includes one flaw echo. We select three central scales

<table>
<thead>
<tr>
<th>a</th>
<th>( j \leq 5 )</th>
<th>( j = 6 )</th>
<th>( j = 7 )</th>
<th>( j = 8 )</th>
<th>( j = 9 )</th>
<th>( j &gt; 9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.33</td>
<td>1.40</td>
<td>22.37</td>
<td>64.13</td>
<td>11.75</td>
<td>0.01</td>
</tr>
<tr>
<td>1.2</td>
<td>0.59</td>
<td>5.60</td>
<td>31.05</td>
<td>53.65</td>
<td>9.32</td>
<td>0.01</td>
</tr>
<tr>
<td>1.4</td>
<td>0.84</td>
<td>9.72</td>
<td>38.23</td>
<td>42.11</td>
<td>7.05</td>
<td>0.01</td>
</tr>
<tr>
<td>1.6</td>
<td>1.17</td>
<td>14.90</td>
<td>47.40</td>
<td>33.04</td>
<td>4.15</td>
<td>0.00</td>
</tr>
<tr>
<td>1.8</td>
<td>1.47</td>
<td>19.22</td>
<td>55.82</td>
<td>22.23</td>
<td>2.30</td>
<td>0.00</td>
</tr>
<tr>
<td>2.0</td>
<td>1.73</td>
<td><strong>22.37</strong></td>
<td><strong>64.13</strong></td>
<td><strong>11.75</strong></td>
<td><strong>0.01</strong></td>
<td><strong>0.00</strong></td>
</tr>
</tbody>
</table>

Bold values indicate the cases for which the energy is most concentrated and the corresponding energy percentages.
To obtain stable statistical results, we test 100 times for each selected central scale, and in each test, the delay of flaw echo is randomly set. The approximate linear relation between the output SNR and the input SNR is shown in Fig. 8. The average SNR improvements reach 19.40, 21.96, and 24.47 dB for the three cases of $j_c$.

As pointed out in [6], the flaw echo can be detected when the output SNR is higher than 4 dB. Fig. 8 shows that, for the central scales $j_c = 8, 9, \text{ and } 10$, the flaw echo can be detected with input SNR above $−15.40, −17.96, \text{ and } −20.47$ dB, respectively. Thus, the SMW-based method is effective in the improvement of noise suppression.

On the other hand, we test the robustness of the proposed method on the SNR improvement for various transmitted signals and wavelet bases, both of which satisfy the concentration rules. Let the central scale $j_c = 9$, and set the input SNR in the interval $[-20, 0]$ dB. The transmitted signal is selected as sym4, sym5, sym6, sym7, and sym8. The wavelet bases are chosen as sym10, sym13, sym15, sym17, and sym20. Table VI shows the average SNR improvement of various combinations of the transmitted signal with the wavelet basis. As Table VI shows, the minimal SNR improvement is 21.82 dB and the maximum SNR improvement is 22.09 dB. Therefore, this experiment verifies the robustness of the proposed method.

As the experimental results show, the proposed SMW is effective in noise suppression and flaw detection. Compared with [6], the proposed method can improve the flaw echo detection ability by about 5 dB in high background noise.

VI. Conclusions

In this paper, we propose the SMW method for UFD in high background noise. SMW overcomes the shortcoming of traditional methods which have difficulty in generating the optimal match because of the constraints of the wavelet. In SMW, the transmitted signal is designed to be a wavelet function to obtain a localized energy distribution of flaw echoes in the WT domain. Three rules for optimal energy matching were proposed by analyzing the energy distribution of the echo signal in the WT domain. Furthermore, the scheme for choosing parameters was put forward for applying the proposed SMW to actual UFD. In addition, a low-rate equivalent DWT structure based on polyphase decomposition was developed, which reduced the hardware cost and further facilitates the practical application of the proposed method. Experimental results were provided to show the validation of optimal matching from energy distribution of flaw echo in the WT domain and the performance of noise suppression. The experimental results show that SMW can efficiently detect flaws under high background noise even for input SNR as low as $−20$ dB.

APPENDIX

Proof of Concentration Rule 2

To facilitate the proof, we first cite a lemma which describes the decay property of a signal $f$ in the WT domain.

**Lemma 1** [4]: If a mother wavelet $\psi(t)$ has a vanishing moment $K$, $K \in \mathbb{Z}^+$, and there is a function $f \in L^2(\mathbb{R})$ which is uniformly Lipschitz $\alpha < K$ over $[\xi_1, \xi_2]$, then there exists $A > 0$ such that

$$\forall (u, s) \in [\xi_1, \xi_2] \times \mathbb{R}^+, \quad |Wf(u, s)| \leq As^{\alpha+1/2}, \quad (A1)$$

where $u$ is the shift parameter, $s$ is the scale parameter, and $Wf(u, s)$ are the wavelet coefficients corresponding to the WT of $f$ on the basis $\{1/\sqrt{s}\psi(t-u)/s\}$, $(u, s) \in [\xi_1, \xi_2]$.

<table>
<thead>
<tr>
<th>Transmitted signal</th>
<th>Wavelet basis</th>
<th>sym10</th>
<th>sym13</th>
<th>sym15</th>
<th>sym17</th>
<th>sym20</th>
</tr>
</thead>
<tbody>
<tr>
<td>sym4</td>
<td></td>
<td>21.82</td>
<td>22.02</td>
<td>21.98</td>
<td>21.87</td>
<td>21.83</td>
</tr>
<tr>
<td>sym5</td>
<td></td>
<td>22.01</td>
<td>22.07</td>
<td>21.96</td>
<td>21.85</td>
<td>21.99</td>
</tr>
<tr>
<td>sym7</td>
<td></td>
<td>22.01</td>
<td>21.95</td>
<td>21.96</td>
<td>22.04</td>
<td>22.07</td>
</tr>
<tr>
<td>sym8</td>
<td></td>
<td><strong>22.09</strong></td>
<td>22.04</td>
<td>22.00</td>
<td>22.01</td>
<td>22.07</td>
</tr>
</tbody>
</table>

Each case was tested 400 times.

Bold values indicate the minimum and maximum SNR improvements.
The proof of concentration rule 2 can be addressed in the following cases: case I, scale \( j < j_c \), and case II, scale \( j > j_c \).

A. Case I: Scale \( j < j_c \)

We give the proof from three aspects: Lipschitz regularity, vanishing moment, and support length. To ease the deduction, the support region of a flaw echo \( s(t - \tau) \) is denoted by \( R_t \) and its length is denoted by \( |R_t| \).

1) Lipschitz Regularity: Let the transmitted signal \( s(t) \) be uniformly Lipschitz \( \alpha_0 \) over \([-\infty, \infty]\), then a flaw echo \( s(t - \tau) \), as a delayed version of \( s(t) \), is also Lipschitz \( \alpha_0 \).

Applying Lemma 1 to DWT, if a basis \( \{\psi_{j,k}\}, j, k \in \mathbb{Z} \) has a vanishing moment \( K > \alpha_0 \), the following relation is satisfied:

\[
\forall j, k \in \mathbb{Z}, \quad |d_{j,k}^\alpha| = \left| \int_{-\infty}^{\infty} s(t - \tau)\psi_{j,k}(t)dt \right| \leq C_1(2^{j})^{\alpha_0 + 1/2}, \\
C_1 \in \mathbb{R}^+.
\] (A2)

It can be found from (A2) that for a basis \( \{\psi_{j,k}(t)\} \) with vanishing moment \( K > \alpha_0 \), when the scale \( j \) decrease, the wavelet coefficients \( d_{j,k}^\alpha \) decays with the degree of \( \alpha_0 + 1/2 \). Thus, if the transmitted signal \( s(t) \) has a large uniformly Lipschitz regularity \( \alpha_0 \), when scale \( j \) decreases (\( j \to 0 \)), the wavelet coefficients have a fast decay.

2) Vanishing Moment: Let \( \psi_{j,k}(t) \) have a vanishing moment \( K_\psi \). \( K_\psi \in \mathbb{Z}^+ \) and its support region is \( R_{j,k} \). Let \( s(t - \tau) \) be uniformly Lipschitz \( \alpha_{j,k} \) in \( R_{j,k} \). The relation \( \alpha_{j,k} \geq \alpha_0 \) can be easily obtained. The smaller the length of \( R_{j,k} \), the larger is \( \alpha_{j,k} \). Given an arbitrary pair \((j_1, k_1)\) satisfying \( j_1 < j_c \) and \( 2^k k_1 \in R_\tau \), the following relation is deduced according to Lemma 1:

\[
\forall j < j_c, 2^k k_1 \in R_{j,k} : \quad |d_{j,k}^\alpha| = \left| \int_{-\infty}^{\infty} s(t - \tau)\psi_{j,k}(t)dt \right| \leq C_2(2^{j})^{\beta_{j,k} + 1/2}, \\
C_2 \in \mathbb{R}^+.
\] (A3)

where \( \beta_{j,k} = \min\{\alpha_{j,k}, K_\psi\} \). Eq. (A3) shows that, if the vanishing moment \( K_\psi \) is large enough, when the scale parameter \( j \) decreases, the wavelet coefficients have decay with degree higher than \( \alpha_0 + 1/2 \).

3) Support Length: Given two wavelet bases: one is \( \{\psi_{j,k}(t)\} \) with vanishing moment \( K_\psi \), and support region \( R_{j,k} \) for the function \( \psi_{j,k}(t) \), and the other is \( \{\hat{\psi}_{j,k}(t)\} \) with the same vanishing moment \( K_\hat{\psi} \) but different support region \( \hat{R}_{j,k} \) for function \( \hat{\psi}_{j,k}(t) \). The lengths of these two support regions satisfy \( |R_{j,k}| < |\hat{R}_{j,k}| \) and \( \hat{R}_{j,k} \subset R_{j,k} \). The wavelet coefficients of \( s(t - \tau) \) on \( \{\psi_{j,k}(t)\} \) and \( \{\hat{\psi}_{j,k}(t)\} \) are denoted by \( d_{j,k} \) and \( \hat{d}_{j,k} \), respectively. Then, for an arbitrary pair \((j_2, k_2)\) satisfying \( j_2 < j_c \) and \( 2^k k_2 \in R_\tau \), we have

\[
\hat{d}_{j,k} < d_{j,k} \leq \hat{d}_{j,k}.
\]

B. Case II: Scale \( j > j_c \)

Because the flaw echo \( s(t - \tau) \) is selected as a wavelet function from the wavelet basis \( \{W_{j,k}(t - \tau), j, k \in \mathbb{Z}\} \), it satisfies the dilation and shift relations. Thus we express the coefficients \( d_{j,k} \) as

\[
d_{j,k} = \int_{-\infty}^{\infty} W(t)(t - \tau)\psi_{j,k}(t)dt \\
= \sum_{i=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2^{j}}} W_{j,k}(u)\psi(u)du \\
= \sum_{i=-\infty}^{\infty} \int_{-\infty}^{\infty} W_{j,k}(u - 2^{-j}\tau)\psi(u)du.
\] (A5)

Eq. (A5) shows a distinct fact: not only can the coefficient \( d_{j,k} \) be thought of as the projection of the echo signal \( W(t - \tau) \) on basis \( \psi_{j,k}(t) \), but it can be thought of as the projection of \( \psi(t) \) on the basis \( W_{j,k}(t - 2^{-j}\tau) \). Therefore, the analysis procedure and the conclusion for Case II are similar to that for Case I, differing only on the suitable signal being \( s(t) \) rather than \( \psi_{j,k}(t) \). Although a higher vanishing moment of a function usually causes longer support length, which will reduce the detection precision when it is selected as a transmitted signal, the requirement of high vanishing moment is not included in the concentration rule 2 for Case I. Based on this analysis, concentration rule 2 for Case II is proven.

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