Spatial-temporal separable filter for adaptive clutter suppression in airborne radar


A spatial-temporal separable filter (STSF) for adaptive clutter suppression in airborne radar is proposed. The STSF coefficients can be efficiently obtained by the proposed bi-iterative algorithm. The STSF performance is illustrated by experiment results on the simulated data and measured data.

Introduction: The optimum space-time adaptive processing (STAP) [1] is essentially a spatial-temporal inseparable filter (STIF), which can most efficiently suppress the wide and strong clutter returns in the airborne phased array radar. However, the optimum STAP usually takes a great deal of training data and has very high computational complexity. Some practical dimension-reduced STAP methods [1–3] have been established for reducing the computational complexity and the training data support. In this Letter, the main objective is to develop an efficient STSF for adaptive clutter suppression in airborne radar. Like many dimension-reduced STAP methods [1–3], the proposed STSF can significanlly decrease computational complexity and the training data support.

Signal model: A \( P \times N \) rectangular antenna array with spacing \( d \) between adjacent elements is assumed to be mounted on an airborne aircraft. Each of the \( N \) column subarrays is synthesised first, and a resulting \( N \)-element uniform linear array is obtained. The airborne radar transmits a coherent burst of \( M \) pulses during a coherent processing interval at a constant pulse repetition frequency \( f_c \). The received data for a range cell can be organised into an \( N \times M \) space-time data matrix

\[
X = \begin{bmatrix}
  x_{1,1} & x_{1,2} & \cdots & x_{1,M} \\
  x_{2,1} & x_{2,2} & \cdots & x_{2,M} \\
  \vdots & \vdots & \ddots & \vdots \\
  x_{N,1} & x_{N,2} & \cdots & x_{N,M}
\end{bmatrix}
\]

Many often-used processors [1–3] can considerably suppress clutter returns in the space-time data matrix \( X \).

Inseparable filter: Data matrix (1) is usually vectorised by stacking each succeeding column beneath the other as \( x = [x_{1,1}, x_{1,2,} \ldots, x_{1,M}, x_{2,1}, x_{2,2}, \ldots, x_{2,M}, \ldots, x_{N,1}, x_{N,2}, \ldots, x_{N,M}] \) lying in \( C_{NM} \). Let the spatial steering vector and the temporal steering vector be \( s_\mathbf{N} = [s_1, s_2, \ldots, s_M]^T \) and \( s_\mathbf{T} = [s_1, s_2, \ldots, s_M]^T \), respectively, then the space-time steering vector [1] is written as \( s = s_\mathbf{N} \otimes s_\mathbf{T} \), where \( \otimes \) denotes the Kronecker product. Let the weight vector \( \mathbf{w} = [w_{1,1}, w_{1,2}, \ldots, w_{1,M}, w_{2,1}, \ldots, w_{N,M}] \) lying in \( C_{NM} \) be similar to data vector \( x \) in form, the space-time processor [1] linearly combines the elements of the data snapshot into a scalar output

\[
y = \mathbf{w}^H x = \sum_{m=1}^{M} \sum_{n=1}^{N} w_{n,m}^* s_{n,m}
\]

The well-known minimum variance distortionless (MVDR) beamformer is determined by the following criterion:

\[
\min_{\mathbf{w}} \{ ||\mathbf{w}^H x||^2 \} = E\left\{ \sum_{m=1}^{M} \sum_{n=1}^{N} w_{n,m}^* s_{n,m} \right\}^2
\]

subject to

\[
\mathbf{w}^H s_{n,m} = \sum_{m=1}^{M} \sum_{n=1}^{N} w_{n,m}^* s_{n,m} = 1
\]

Thus, \( w_{n,m}(n = 1, \ldots, N \) and \( m = 1, \ldots, M \) represent the STIF coefficients for adaptive clutter suppression, which has \( NM \) degrees of freedom (DOFs). Once the solution of (3) is found, the STIF for adaptive clutter suppression is obtained. However, if the number of training data is set as \( L \geq 2MN \), which is often very large, solving (3) will take computational complexity \( O(LNM^2) + O(NM^3) \), which is very high.

Separable filter: In the following, we design a separable filter for adaptive clutter suppression. According to the well-known separation of variables, let coefficient \( w_{n,m} \) be expressed as separable form \( w_{n,m} = u_n v_m \) that is inserted into (3), then we obtain the following biquadratic cost function:

\[
\min_{u_n, v_m} \left\{ \sum_{m=1}^{M} \sum_{n=1}^{N} u_n^* v_m^* s_{n,m} \right\}^2 = E\left\{ ||\mathbf{u}^H \mathbf{X} \mathbf{v}||^2 \right\} \quad (4a)
\]

subject to

\[
\sum_{m=1}^{M} \sum_{n=1}^{N} u_n^* v_m^* s_{n,m} = (\mathbf{u}^H \mathbf{s}_1)(\mathbf{v}^H) = 1 \quad (4b)
\]

where \( \mathbf{u} = [u_1, \ldots, u_M]^T \) is called the spatial filter-coefficient vector and \( \mathbf{v} = [v_1, \ldots, v_N]^T \) called the temporal filter-coefficient vector. Moreover, let the constraint condition in (4) be separated into two constraint conditions as shown in (5), and we get the following optimisation problem:

\[
\min_{u_n, v_m} \left\{ ||\mathbf{u}^H \mathbf{X} \mathbf{v}||^2 \right\} \quad \text{subject to} \quad \mathbf{u}^H \mathbf{s}_1 = 1 \quad \text{and} \quad \mathbf{v}^H \mathbf{s}_1 = 1 \quad (5)
\]

It is obvious that the two constraint conditions in (5) can directly deduce that in (4). The above problem includes \( N \times N \) independent variables or DOFs.

By utilising the Lagrange multiplier method, (5) can be further converted into the following biquadratic cost function without constraint:

\[
\min_{u_n, v_m, \mu_1, \mu_2} \left\{ ||\mathbf{u}^H \mathbf{X} \mathbf{v}||^2 \right\} + \mu_1 (\mathbf{u}^H \mathbf{s}_1 - 1) + \mu_2 (\mathbf{v}^H \mathbf{s}_1 - 1) \quad (6)
\]

where \( \mu_1 \) and \( \mu_2 \) are Lagrange multipliers. According to the idea in [4], we propose the following bi-iterative algorithm (BIA) for solving (6). First, fix \( v \) and let the gradient of \( J(u, v, \mu_1, \mu_2) \) with respect to \( u \) and \( \mu_1 \) be equal to zero, and we easily obtain an estimated spatial weight vector \( \mathbf{u} = \mathbf{R}_1^{-1} \mathbf{s}_1^* / \mathbf{R}_1 \), where \( \mathbf{R}_1 = E(\mathbf{Xv} \mathbf{v}^H) \).

Secondly, fix \( u \) and let gradient \( J(u, v, \mu_1, \mu_2) \) with respect to \( v \) and \( \mu_2 \) be equal to zero, and we get an estimated temporal weight vector \( \mathbf{v} = \mathbf{R}_2^{-1} \mathbf{s}_2^* / \mathbf{R}_2 \), where \( \mathbf{R}_2 = E(\mathbf{X}^H \mathbf{u}^H \mathbf{X}) \).

Based on the above analysis, the efficient BIA follows.

Let the initial value \( v(0) = s_2^* / ||s_2||^2 \). For \( k = 1, 2, \ldots \) until \( ||\mathbf{u}(k) - \mathbf{u}(k-1)|| < \varepsilon \) (the tolerance error \( \varepsilon \))

(a) perform \( u(k) = \mathbf{R}_1^{-1}(k) s_1 / [\mathbf{R}_1^{-1}(k) s_1] \), where \( \mathbf{R}_1(k) = E(\mathbf{Xv} (k-1) \mathbf{v}^H) \);

(b) calculate \( v(k) = \mathbf{R}_2^{-1}(k) s_2 / [\mathbf{R}_2^{-1}(k) s_2] \), where \( \mathbf{R}_2(k) = E(\mathbf{X}^H \mathbf{u}(k) \mathbf{u}(k)^H) \).

Interestingly, it can be shown by the analysis method used in [4] that this algorithm is convergent.

Fig. 1 IF against iteration number

Computational complexity: It is seen from two correlation matrices \( \mathbf{R}_1 = E(\mathbf{Xv} \mathbf{v}^H) \mathbf{X} \) and \( \mathbf{R}_2 = E(\mathbf{X}^H \mathbf{u} \mathbf{u}^H) \mathbf{X} \) that the number of training data should be larger than \( 2 \max[N,M] \). In addition, it can also be shown by experiment that, if \( \varepsilon = 0.001 \), the above BIA will converge in about six iterations, as shown in Fig. 1. If the number of training data is selected as \( L \), it can be computed that the multiplication and division number in the BIA is about \( 6(LM^2 + N^2) + 2(M^2 + N^2) / \max[N,M] + 3 \). Since there generally are \( L \ll L \) and \( \max[N,M] \ll MN \), STSF is also a dimension-reduced STAP technique.

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Numerical experiments: We set the parameters as follows. Let \( P = 16 \), \( N = 24 \), \( M = 24 \), \( f_r = 2000 \text{ Hz} \) and \( d = 0.115 \text{ m} \). The radar wavelength is taken as \( \lambda = 0.23 \text{ m} \), the platform velocity is \( V = 115 \text{ m/s} \), platform altitude is set as \( H = 6 \text{ km} \), and the clutter-noise-ratio (CNR) is denoted by \( \text{CNR} = 60 \text{ dB} \). Assume that the array is side-looking and the direction of the mainbeam is 90°. Let the variance of both the element’s amplitude and phase errors be 2%.

Fig. 1 shows the improvement factor (IF) against the iteration number, where IF denotes the ratio of output SCNR (signal-to-clutter-plus-noise-ratio) to the input SCNR, and Doppler channel 9 is situated in the mainlobe clutter region. It can be seen from Fig. 1 that the BIA can converge very well after six iterations in the mainlobe clutter region. Fig. 2 shows the performance comparison between the STSF, FA [2] and EFA [2]. It can be seen that the performance of STSF is significantly better than that of FA and may be slightly better than that of EFA. Especially, the STSF can increase IF in the mainlobe clutter region.

![Fig. 2 Curves of IF against normalised Doppler frequency \( f_d/f_r \)](image)

Here, MCARM [5] data from acquisition 575 is processed in this experiment, where 11 azimuth channels and the first 32 pulses are employed, and a simulated target located in range bin 375 with 186 Hz of Doppler is injected at broadside. For performance comparison, the results obtained by STSF, FA and EFA are plotted in Fig. 3. It can be seen that the target peak power relative to average residual clutter power is 30.4 dB for STSF, 24.2 dB for EFA and 19.5 dB for FA. Thus, the STSF achieves relatively good performance at the improvement of an output SCNR and corresponding detection probability under a constant probability of false alarm.

![Fig. 3 Curves of normalised output power against range bin](image)

Conclusion: This Letter develops the STSF for adaptive clutter suppression in airborne radar. Like some existing dimension-reduced STAP methods [1–3], the proposed STSF can also achieve good performance at the cost of less computational complexity and smaller sampling data support.

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